

## Strateg -proof cardinal decision schemes

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As was first pointed out to us by John Hegeman,<sup>1</sup> the proof of Claim 3, Theorem 1, in Dutta et al. (2007) is not correct, since it is based on interchanging two limits which is not justified without, for instance, a continuity assumption.

In this note we first give an alternative proof of  $\lambda_j \leq \lambda'_j$  (notations as in Dutta et al. (2007)).

To show this assume, to the contrary,  $\lambda_j > \lambda'_j$ . By Claim 2 we can take  $u_1$  with  $\tau(u_1) = a_j$  and—for simplicity— $u_1(a) = 0$  for all  $a \neq a_j$ . Let  $0 < \varepsilon < \frac{1}{2}(\lambda_j - \lambda'_j)$  and let  $\eta_2$  be so small that  $|\varphi_j(u_1, u_{kj}^{\eta_2}) - \lambda'_j| < \varepsilon$ . Observe that agent 1's utility in this profile is equal to  $\varphi_j(u_1, u_{kj}^{\eta_2})$ .

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Next, take  $\eta_1$  so small that  $|\varphi_j(u_{jk}^{\eta_1}, u_{kj}^{\eta_2}) - \lambda_j| < \varepsilon$ . This is possible in view of Claim 2 (with the roles of the agents there reversed). Then according to  $u_1$  agent 1's utility is now  $\varphi_j(u_{jk}^{\eta_1}, u_{kj}^{\eta_2})$ , but  $\varphi_j(u_{jk}^{\eta_1}, u_{kj}^{\eta_2}) > \varphi_j(u_1, u_{kj}^{\eta_2})$ . This violates strategy-proofness.

Unfortunately, at this moment we do not know how to prove the reverse inequality  $\lambda_j \geq \lambda'_j$  and, thus, Claim 3, without making additional assumptions. One possibility would be to extend the set of admissible utility functions by dropping the requirement that there be a unique top alternative and assume continuity of the **CDS**  $\varphi$ . Another possibility is to strengthen the unanimity condition by requiring that if every agent in a preference profile has the same two top alternatives, then all other alternatives should receive zero probability. A third possibility is to impose, additionally, the following requirement on  $\varphi$ , which is a kind of unanimity:

(\*) For all admissible profiles  $u$  and all  $a_j \in A$ , if  $u_i(a_k) \geq u_i(a_j)$  for all  $a_k \in A \setminus \{a_i\}$  and  $i \in N$ , then  $\varphi_j(u) = 0$ .

In other words, if the agents have a common bottom alternative, then that alternative should receive zero probability. We will prove  $\lambda_j \geq \lambda'_j$  under this additional assumption (\*). To the contrary, assume  $\lambda_j < \lambda'_j$ . Let  $0 < \varepsilon < \frac{1}{2}(\lambda'_j - \lambda_j)$ . By Claim 2, we may assume that the utility functions under consideration have a common bottom alternative  $b$ . Also by Claim 2, we may assume that  $u_{jk}^{\eta_1}$  satisfies  $u_{jk}^{\eta_1}(a) = 1 - \eta_1 - \alpha(\eta_1)$  for all (if any)  $a \neq a_j, a_k, b$ , with  $\alpha(\eta)$