Abstract

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Key Words:
JEL Classification:

1 Introduction

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2 The Model

$$= \int_0^{\infty} {}^{-\rho t} \frac{{}^{1-\sigma} - 1}{1 -}$$

0

0

0

$$+ \dot{} + \dot{} + \dot{} = (-) + - -$$

$\dot{} \equiv \frac{dx}{dt}$

$\equiv$ fi fl

$\equiv \dot{}$ fl

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$$+ \dot{} \leq$$

fi = +

$$+ \dot{} + \dot{} = (-) + - -$$

(0) = $_0$ 0 (0) = $_0$ 0

k

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$$^{-\sigma} = \quad + $$

$$ = $$

$$ + \quad = \quad _k$$

$$\dot{} = \quad (\quad + \quad - \quad)$$

$$\dot{}_k = \quad _k -$$

$$\lim_{t\to\infty} {}^{-\rho t} \quad _k \quad = \lim_{t\to\infty} {}^{-\rho t} \quad = 0$$

$$\dot{} = \quad -$$

(5) (7) (9)

$$ = \dot{-} = \frac{1}{} \left(\frac{}{_k} \quad - \quad \right)$$

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$$- = \quad - \dot{-}$$

$$- = - = — = - =$$

(8) (9) (11)

$$- = \frac{k}{k} = -$$

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$$(- ^*) + (- *) = {}_i (- ^*) =$$

i

$* * *$

$= 1 = 0$

${}_R \geq 0$

$$= ^* + {}_R (- ^*)$$

${}_R 1$ ${}_R 1$ ${}_R \neq 1$

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$= 0$ $= 1$ $_{\mu} \geq 0$

$$= {}^{*} + {}_{\mu}(- {}^{*})$$

$_{\mu}$ 1 $_{\mu}$ 1

$_{\mu} = 0$

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2.2 Equilibrium Analysis

2.2.1 *Money Growth Rules*

$$' = (-)$$

(10) (17) fl

$$= \frac{1}{1 - {}_{\mu}} ({}^{*} - {}_{\mu} {}^{*} - +)$$

$\equiv$

$$\frac{'}{} = \left(\frac{}{} + - \frac{+}{} \right)$$

$\equiv {}_{k}$ (6) (7) fi

$$= \quad - 1$$

$$\dot{\bar{}} = \left[\; - - - 1 - \frac{1}{1 - \quad_\mu} \left(\; ^* - \quad_\mu \; ^* - \quad + \; \right) \right]$$

(18) (20)

(19) (21)

Comparative Statics (19) (21)

$$(\bar{} \; \bar{})$$

$$(\bar{})^2 - \;_1 \bar{} + \;_2 = 0$$

$$\bar{} = - + \left(1 - \frac{1}{=} \right)$$

$$_1 \equiv \frac{\rho}{\tilde{\sigma}} + 1 + \frac{\mu^* - \gamma_\mu \pi^*}{1 - \gamma_\mu} \quad _2 \equiv \left(\frac{1 - \tilde{\sigma}}{\tilde{\sigma}} \right) \qquad \tilde{} \equiv \left(1 - \quad_\mu \right)$$

$$\bar{} \qquad ^*$$

$$\frac{\bar{}}{^*} = \frac{\bar{}}{2 \left(1 - \quad_\mu \right) \left(\bar{} - \;_1 \; 2 \right)}$$

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$$= \frac{1}{} \left[= - \; \right]$$

$$\frac{}{^*} = - \frac{}{(\bar{})^2} \frac{\bar{}}{^*} = - \frac{}{2 \; ^* \bar{} \left(1 - \quad_\mu \right) \left(\bar{} - \;_1 \; 2 \right)}$$

$$\bar{} = \frac{_1 \pm \sqrt{\Delta}}{2}$$

$\Delta \equiv (\ \ _1)^2 - 4\left(\frac{1-\tilde{\sigma}}{\tilde{\sigma}}\right)$ ˜ 0 ˜ 1

−

− $_1$ 2

$\bar{}_1$ $\bar{}_2$

$$\bar{}_1 \quad \frac{1}{2} \quad \bar{}_2 \quad (\bar{}_2) \quad (\ _1) \qquad \frac{(\bar{}_2)}{*} \quad 0 \quad \frac{(\bar{}_1)}{*}$$

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$$\bar{} = \bar{} - \frac{}{} - 1$$

$\bar{}$ $\bar{}$

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$\bar{}$

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$\bar{}$

$$\frac{(1-\tilde{\sigma})\,A}{\rho} > 1 + \mu^*$$

$0 < \tilde{\sigma} < 1.$

$$= {}^{-}\left[1 + \frac{}{(\,\bar{}\,)^2}\right] + {}^{-} \quad 0$$

$$\det \qquad \dot{} = 0$$

$$\det = 2\bar{}\,(\bar{} - {}_1\; 2)$$

$$\det \qquad \tilde{} \quad 1$$

$$\bar{} \geq 0$$

$$\det \qquad \tilde{} \quad 0$$

$0 \quad \tilde{} \quad 1 \;\; \det \quad 0 \; (\quad 0) \qquad \bar{}_1 \; (\bar{}_2) \qquad \bar{}_1 \quad \bar{}_2$

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Proposition 1 *In the Ak model with feedback money growth rule, we have non-superneutrality in the growth-rate sense. When* $\tilde{} \quad 1$ *or* $\tilde{} \quad 0$ *there exists a unique balanced growth path equilibrium that is locally determinate and faster money growth retards (promotes) economic growth in the former (latter) case. When* $0 \quad \tilde{} \quad 1$ *then dual BGP equilibria may emerge in which the low-growth equilibrium is locally determinate where faster money growth lowers economic growth, and the high-growth equilibrium is indeterminate where faster money growth raises economic growth.*

Comparative Statics (19) (33)

$$(\bar{\ } \ )$$

$$_3(\bar{\ })^2 + \Omega\bar{\ } - \quad = 0$$

$$\bar{\ } = - + \quad \left(1 - \frac{1}{-}\right)$$

$$_3 = \left(1 - \frac{1}{\gamma_R}\right) \qquad (34)$$

$$\bar{\ } = \frac{-\Omega \pm \sqrt{\Delta}}{2 \ _3}$$

$\Delta \equiv \Omega^2 + 4\left(1 - \frac{1}{\gamma_R}\right) \quad 0$

$_R \quad 1$

$\bar{\ }$

$\bar{\ } = \left(-\Omega + \sqrt{\Delta}\right) \ 2 \ _3$

$_R \quad 1$ $\bar{\ }$ $\bar{\ }_1$ $\bar{\ }_2$

$\bar{\ }_1$ $\bar{\ }_2$

*

$$\frac{\quad}{\ ^*} = -\frac{\quad}{(\bar{\ })^2}\frac{\bar{\ }}{\ ^*}$$

$\frac{d\bar{p}}{dR^*}$

Lemma 2 $|\Omega|$ ( ) $\sqrt{\Delta}$ *for passive (active) interest rate rules.*

Proof. *From the definition of* Δ *and notice that* $1 - \frac{1}{\gamma_R}$ ( ) 0 *for the case of active (passive) interest rate rules, and the result follows.* ■

$\bar{\ } = \left(-\Omega + \sqrt{\Delta}\right) \ 2 \ _3$

$$\frac{\bar{\ }}{\ ^*} = \frac{_3 - 1}{2 \ _3}\left[1 - \frac{\Omega}{\sqrt{\Delta}}\right] \quad 0 \qquad \frac{\quad}{\ ^*} \quad 0$$

$\Delta > 0$

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$$\frac{\bar{}_1}{^{*}} = \frac{_3 - 1}{2\,_3}\left[1 + \frac{\Omega}{\sqrt{\Delta}}\right] \quad 0 \quad \frac{\bar{}_2}{^{*}} = \frac{_3 - 1}{2\,_3}\left[1 - \frac{\Omega}{\sqrt{\Delta}}\right]$$

$$\frac{(\bar{}_2)}{^{*}} \quad 0 \quad \frac{(\bar{}_1)}{^{*}}$$

$\bar{}_1$ $\bar{}_2$ fi fi

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$$^{*} = \; - \; _R \; + \; _R \; ^{*}$$

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(33)

$$^{*} = \left[1 - \quad _R \left(1 + \frac{}{\;^2}\right)\right] \; -$$

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$_R$ 1 ﬂ

$_R$ 1

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Local Dynamics

(33)

$$\dot{} = \;(\; - \bar{\;})$$

$$\equiv - \left[\left(1 - \frac{1}{\gamma_R}\right) + \frac{}{(\bar{\;})^2}\right]$$

(33) $_R$ 1 0

Lemma 3 $sgn\{\ \} = \left\{\frac{2}{\Omega} - \;\right\} = sgn\left\{\frac{\sqrt{\Delta}\left[\sqrt{\Delta} \pm \Omega\right]}{2\Omega\left(1 - \frac{1}{\gamma_R}\right)}\right\}$

Proof. *We first note that can be written as* $\left(\frac{\Omega}{=}\right)\left(\frac{2}{\Omega}-\frac{}{}\right)$ *by (34). Thus, we obtain the first equality. To obtain the second equality, we use (36) to substitute away* $\bar{}$ *and yield* $\frac{\Delta \pm \Omega\sqrt{\Delta}}{2\Omega\left(1-\frac{1}{\gamma_R}\right)}$. *Factorize* $\sqrt{\Delta}$ *out gives the result.* ■

fi $\bar{}$ Ω 0

Ω $\sqrt{\Delta}$ $_R$ 1

( )0

$\bar{}_1(\bar{}_2)$ fi

Proposition 4 *In the Ak model with active interest rate rule* ($_R$ 1)*, there is a unique BGP equilibrium and it is locally determinate. When interest rate rule becomes passive* ($_R$ 1)*, dual BGP equilibria emerge. The high-growth equilibrium* ($\bar{}_2$) *is locally determinate while the low-growth equilibrium* ($\bar{}_1$) *is indeterminate. Also, at the high- (low-) growth equilibrium, an increase in the interest rate target accelerates (retards) economic growth.*

(7) (9)

$$\frac{\dot{}}{} = \left[(-) - \frac{}{}\right]$$

0

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3 Equivalence on Monetary Policies

Definition 5 *Two monetary policy regimes are equivalent if*

1. both policy rules yield the same BGP equilibria, and

2. both the BGP equilibria exhibit same equilibrium dynamics, and

3. the comparative statics results are qualitatively equivalent at the determinate BGP equilibrium.

$$(\bar{\ })^2 - \left(1 + \frac{}{\sim} + \frac{{}^{*} - \ \mu\ {}^{*}}{1 - \ \mu}\right)^{-} + \left(\frac{1 - \tilde{\ }}{\sim}\right) \quad = 0$$

$$(\bar{\ })^2 - \left(1 + \frac{\ ^*}{1 - \ _R} - \frac{\ _R\ ^*}{1 - \ _R}\right)\bar{\ } + \left(\frac{\ _R}{1 - \ _R}\right) \quad = 0$$

$$\frac{1 - \tilde{\ }}{\tilde{\ }} = \frac{\ _R}{1 - \ _R}$$

$$\frac{\ }{\tilde{\ }} + \frac{\ ^* - \ _\mu\ ^*}{1 - \ _\mu} = \frac{\ ^* - \ _R\ ^*}{1 - \ _R}$$

$$\ _R = 1 - \ \left(1 - \ _\mu\right) = 1 - \tilde{\ }$$

$_R \quad 0$ $\tilde{\ } \quad 1$ $\tilde{\ } \quad 0$

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$$\ ^* = \ ^* + [\ + (1 - \ )\ ^*]$$

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$_R \quad 1$

$_\mu \quad 1$

$_\mu = 0$

$_R \quad 1$ $_\mu \quad 1$

$\tilde{\sigma} > 1$

$\sigma \geq 1$ $\gamma_R > 0$ $\gamma_\mu < 0$

* *

$\tilde{}$ 0

$$(\frac{\bar{}}{^{*}}) = (\frac{\bar{}}{^{*}})$$

$\tilde{}$ 0 $_{\mu}$ 1

0

$\tilde{}$ 1 1 $_{\mu}$ $1-1$

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$$\frac{\bar{}_{2}}{^{*}} \quad 0 \quad \frac{\bar{}_{1}}{^{*}}$$

$\bar{}_{1}$ $\bar{}_{2}$ $=$

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Proposition 6 *Consider the following two types of monetary policy feedback rules:*

1. money growth rules: $= {}^{*} + {}_{\mu}(- {}^{*})$

2. interest rate rules: $= {}^{*} + {}_{R}(- {}^{*})$

Then an active interest rate rule (${}_{R}$ 1*) is equivalent to an active money growth rule (* ${}_{\mu}$ 1*) where a unique determinate BGP equilibrium emerges and an increase in the monetary policy target improves economic growth performance. On the other hand, under passive interest rate rules (* ${}_{R}$ 1*) and passive money growth rules (*1 ${}_{\mu}$ $1-1$ *), real indeterminacy can occur. Also, we are unable to establish equivalence between passive monetary policy rules because the comparative statics results at the determinate BGP equilibrium are not qualitatively equivalent.*

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$1 \quad \mu = 0$

$1 - 1 \quad 0$

μ

$1 \quad \mu = 0 \quad 1 - 1$

Corollary 7 *If the intertemporal elasticity of substitution in consumption is greater than unity (1), constant money growth rules cannot mimic any feedback interest rate rules.*

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$= \quad _R = 0 \qquad = \quad ^*$ fi

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$= 1 + \quad ^*$

$= 0$

$^* = \quad ^* - \frac{}{1 + \quad ^*}$

$_\mu = 1 - 1$

$\gamma_\mu = 1 - 1/\sigma$

fl $\sigma > 1$

$$ {}^{*} + \frac{(1 - \quad)}{1 + \quad {}^{*}} = \quad + \quad {}^{*} $$

$= 1$ $\qquad\qquad$ $_{\mu} = 0$

$$ \bar{\ } = 1 + \quad {}^{*} = 1 + \quad + \quad {}^{*} $$

$$ \bar{\ } \quad {}^{*} = \bar{\ } \quad {}^{*} = 1 $$

Proposition 8 *When the felicity function is logarithmic (* $= 1$*), then nominal interest rate pegging policies (* $_{R} = 0$*) are equivalent to constant money growth rules (* $_{\mu} = 0$*).*

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4 Concluding Remarks

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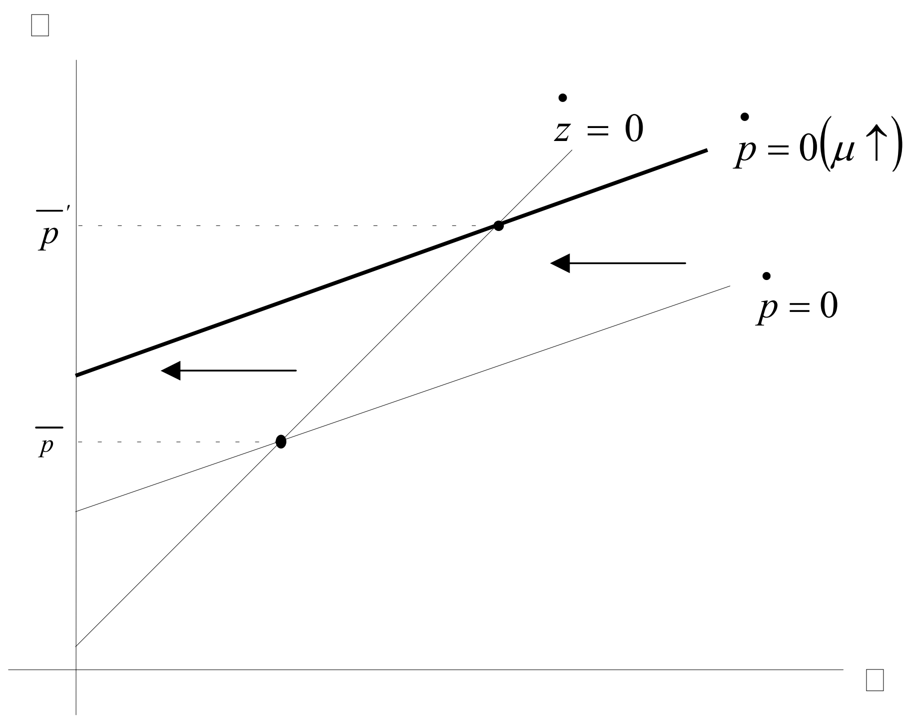
Journal of Money, Credit and Banking



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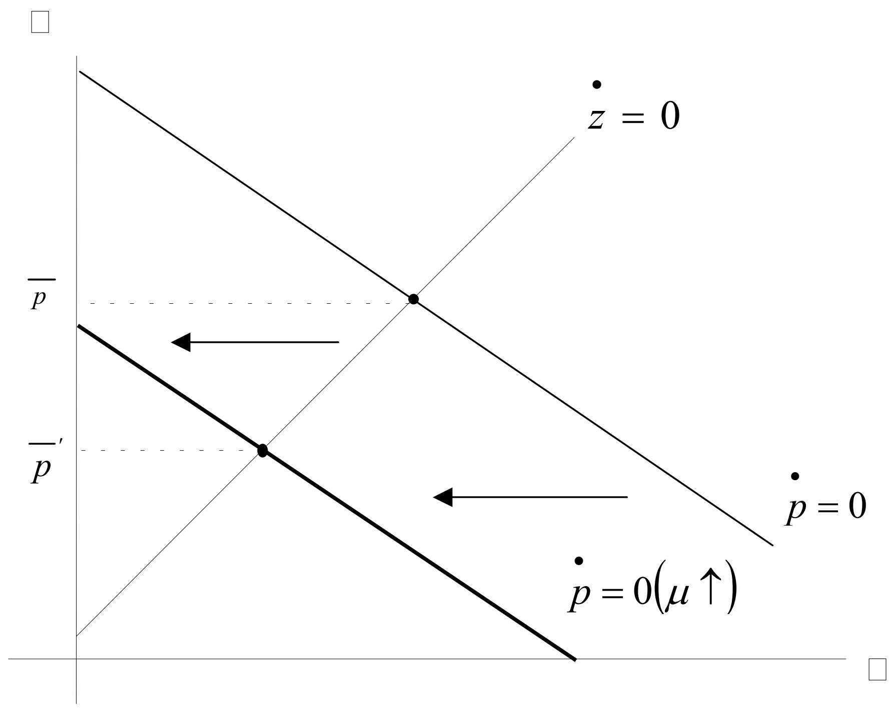

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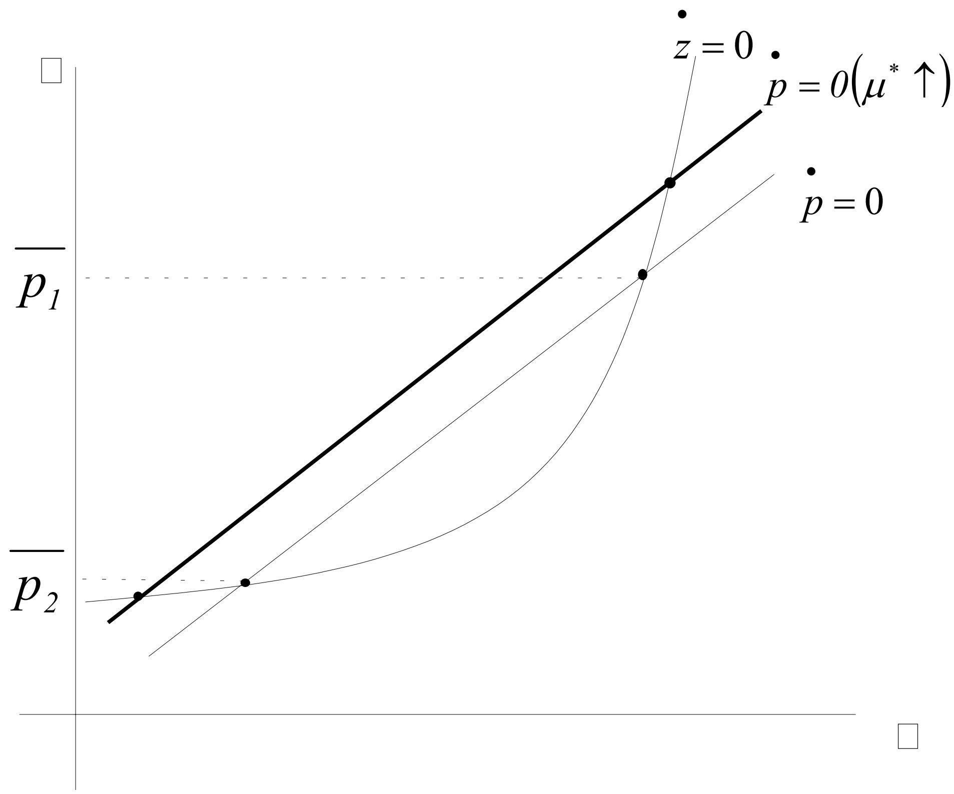

2 μ* $0 < \tilde{\sigma} < 1$

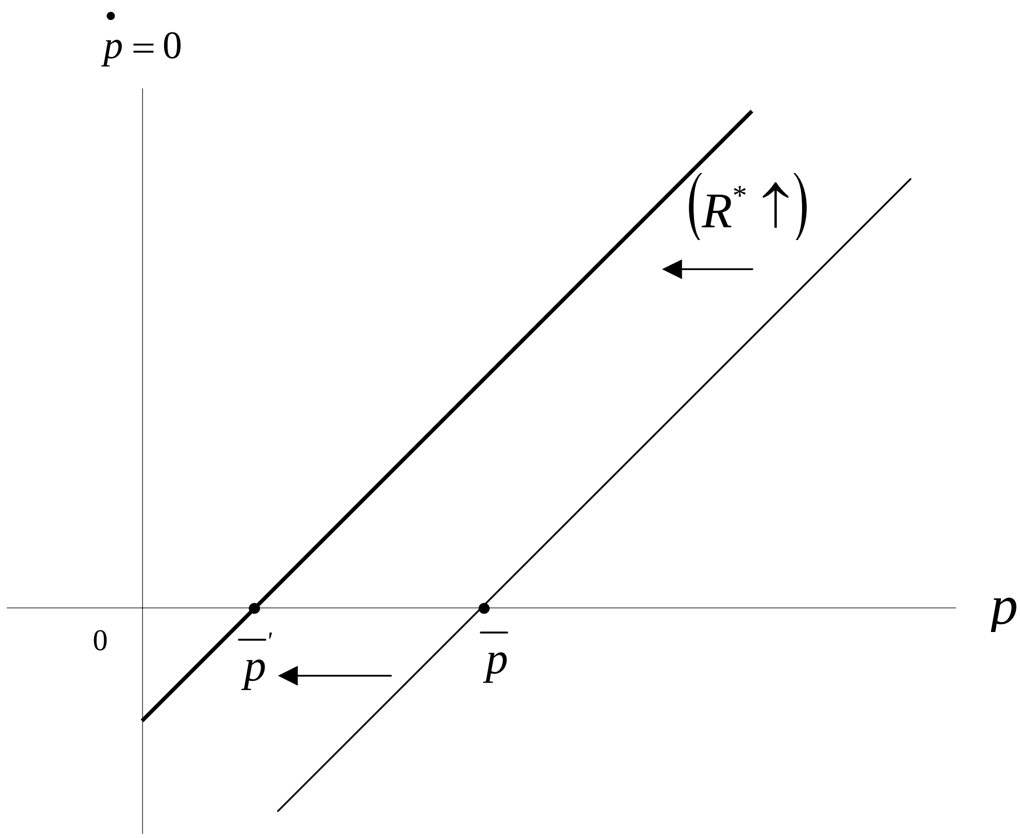

Figure 3 Comparative Statics for an increase in *R** under active interest rate rule