Asset Returns with Earnings Management*

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December 2008

Job Market Paper

Abstract

The paper investigates stock return dynamics in an environment where executives have an incentive to maximize their compensation by artificially inflating earnings. A principal-agent model with financial reporting and managerial effort is embedded in a Lucas asset-pricing model with periodic revelations of the firm's underlying profitability. The return process generated from the model is consistent with a range of financial anomalies observed in the return data: volatility clustering, asymmetric volatility, and excessive idiosyncratic volatility.

Keywords: Earnings management, Stock returns, Financial anomalies, Volatility clustering, GARCH, Optimal contract

JEL Classifications: D82, D83, G12, G14

*The author gratefully acknowledges Eric Young and Chris Otrok for their invaluable inspiration and constant support. A special debt of gratitude is owed to Toshihiko Mukoyama for instructive advice. The author is also greatly indebted to Stan Baiman, Sean Campbell, Mark Carey, Antonio Falato, Pingyang Gao, Jon Glover, Marvin Goodfriend, Rick Green, Nathan Larson, Rick Lambert, Pierre Liang, Leo Martinez, Patrick McCabe, Yohei Okawa, Michael Palumbo, Edward Simpson Prescott, Cathy Schrand, Steven Sharpe, Xuan Tam, Ro Verrecchia, Missaka Warusawitharana, and John Weinberg for their helpful comments. Earlier versions of this paper were presented at the Federal Reserve Board, the Federal Reserve Bank of Richmond, Midwest Theory Conference, Economics Department and Darden School of Business at the University of Virginia, the Wharton School of the University of Pennsylvania, and Tepper School at Carnegie Mellon University. The author would like to thank the seminar participants for numerous useful suggestions. The author thanks the University of Virginia, John M. Olin Foundation, and Federal Reserve Board Dissertation Internship program for financial support. All errors are solely the author's.

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1 Introduction

The primary role of financial reporting is to provide corporate executives with a credible means of communicating their private information on firms' performance to external shareholders. This role can become entangled with executives' desire to use financial reports, especially bottom-line earnings, to pursue their own financial interests. Such motives give rise to the phenomenon of *earnings management*, which is defined as intentional manipulation of reported earnings by knowingly choosing accounting methods and estimates that do not accurately reflect the firm's underlying fundamentals. Empirical evidence suggests that earnings management behavior is pervasive.[1]

In the wake of the massive financial scandals of the early 2000s, the integrity of financial reporting and consequences of earnings management have received increased academic attention and regulatory scrutiny. Distorted information flow can engender substantial economic costs. On average, stock returns fall by about 10% on the days around earnings restatement announcements.[2] Figure 1, reproduced from Wu (2002), documents how stock returns react to restatements.[3] Earnings management also distorts the allocation of capital.[4] Losses to the shareholders are readily apparent. To minimize the economic costs of misreporting, academics, practitioners, and regulators have called for corporate governance reforms to strengthen shareholder power. These efforts, however, do not render the study of financial misreporting irrelevant. Recent research has documented a significant upward trend in the number of restatement announcements over time (See GAO, 2002 and GAO, 2006).

Substantial literature has been devoted to the empirical characterization of earnings management behavior; yet comparatively little is known about how earnings management affects asset returns. This intentional manipulation of financial information must be reflected in the pricing of stocks, because investors are supposed to know what to infer from financial

[1] See Loomis (1999) and McKee (2005).

[2] Turner et al. (2001) report negative market adjusted returns of −12.3% over an eight-day window. Palmrose et al. (2004) document an average abnormal returns of about −9% over a two-day announcement window. Wu (2002) shows that the market reacts negatively with over −11% cumulative abnormal returns during a three-day window.

[3] I thank Min Wu for providing Figure 3 of Wu (2002), which is reproduced as Figure 1 in the current paper.

[4] See Burns and Kedia (2006) and Kedia and Philippon (2007).

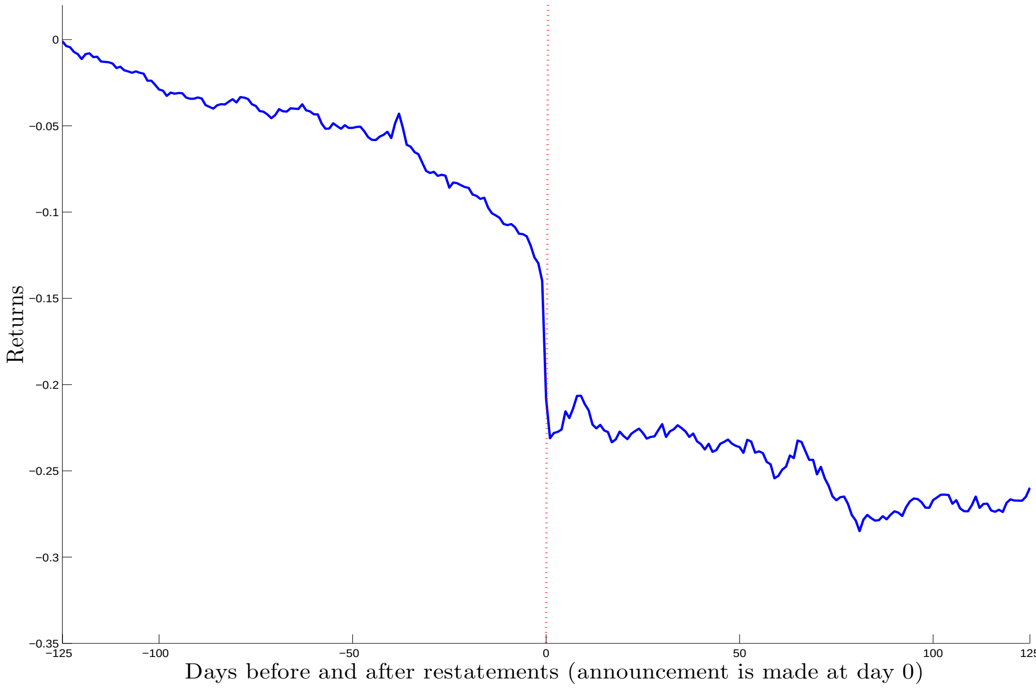

Figure 1: Cumulative abnormal returns around restatements: day (-125,+125)
Source: Wu (2002)

reports when they value the stock of a firm. The objective of the present study is to analyze the implication of earnings management strategies for stock return dynamics.

I conduct this exercise within a Lucas asset-pricing model that is standard in all aspects, except that the investors hire a manager to operate the firm and report the firm's earnings. In particular, a principal-agent model with managerial reporting incentives and productive effort choices is embedded in a simple variant of the Lucas asset-pricing model. As in the standard Lucas asset-pricing model, a group of investors is the representative owner of an apple orchard. The infinitely-lived trees (firms) produce perishable apples (earnings) each period, and the harvest varies from period to period depending on the weather. In contrast with the standard Lucas model, the process of apple production is not entirely exogenous in the current paper. The manager exerts an unobserved effort that affects the production, and possibly has discretion over the quantity of apples reported to the investors. The reported earnings are paid to the investors as dividends. The investors engage in a (single-period) contractual relationship with a newly hired manager in every period, and pay the manager a fraction of the reported earnings as compensation. The key feature I focus on here is the manager's ability to manipulate earnings reports. Earnings management occurs in the model when the reported apple harvest (earnings) differs from the true amount.[5]

There are periodic investigations concerning the underlying true earnings of the firm. In the final period of each auditing cycle, the uncertainty about true earnings is resolved, and the investors bear monetary penalties in the event that earnings management is detected. The investors are assumed to be risk-neutral; thus the price of the firm in each period is given by the discounted expected future dividends net of the labor wage and the fines associated with earnings management.

The return sequences generated from the model mimic a set of stylized facts in stock return data. First and foremost, the model returns exhibit volatility clustering. Because earnings management patterns vary with underlying true performance, certain levels of earnings lead to higher frequency of earnings restatements than others, creating larger swings in the return sequence. Return volatility becomes state-dependent in the model. As the state (that is, actual earnings) exhibits persistence over time, return volatility is time-varying and persistent. In addition, the possibility of earnings management creates a range of reports that are associated with belief revision and intense suspicion of financial misreporting. The anticipation of restatements increases uncertainty and hence volatility. The volatility persists as reported earnings persist. Although the conditional heteroskedasticity observed in many financial markets has led to ARCH and GARCH models that are intensively used in analyzing stock returns, the underlying microeconomic motives are still not well understood. This paper presents the persistence in earnings management behavior as a likely source of the persistence in stock return volatility.

The model data capture another stylized fact in the finance literature: asymmetric volatility in stock returns. The mechanism is twofold. First, earnings management goes hand-in-hand with a weak economic performance, due to stronger financial incentives to inflate earnings when the performance is weaker. Because current low earnings lead to more

[5]The modeling technique presented here bears some similarities with Shorish and Spear (2005). The similarities and differences between their paper and this paper will be discussed later in this section.

frequent future earnings manipulation and resultant drastic consequences, low returns lead to high volatility in subsequent returns. Second, earnings reports at the lower end of the range are viewed as symptomatic of intentional misstatement. The inference of earnings management reduces the current price and increases the uncertainty over subsequent outcomes, thereby intensifying asymmetric volatility.[6] The existing literature on asymmetric volatility falls into two categories: leverage effect proposed by Black and Scholes (1973), Merton (1974), and Black (1976) and volatility feedback effect put forward by French et al. (1987) and Campbell and Hentschel (1992). However, Christie (1982) and Schwert (1989) find that the leverage effect is too small to account for the asymmetry in volatility, and Campbell and Hentschel (1992) find that the volatility feedback effect normally has little impact on returns. This paper shows that a mechanism exists for earnings management to generate the observed asymmetric behavior in stock returns. The calibration results further suggest that this channel can be quantitatively important.

Last but not least important, as earnings management becomes more likely in the model, asset returns exhibit greater volatility. The dramatic consequence of restatement announcements generates active fluctuations in the return sequence and thus intensifies return volatility. This work adds to a growing literature that studies individual stock return volatility. Campbell et al. (2001) document that the level of average stock return volatility increased considerably from 1962 to 1997 in the United States. Furthermore, most of this increase is attributable to idiosyncratic stock return volatility as opposed to the volatility of the stock market index. Rajgopal and Venkatachalam (2007) explore whether deteriorating financial reporting quality, as measured by earnings quality and dispersion in analyst forecasts of future earnings, can plausibly explain the increase in idiosyncratic volatility over the past four decades. Their results from cross-sectional and time-series regressions indicate a strong as-

[6]Rogers et al. (2007) empirically document that strategic disclosure, defined as the reporting of good news and the withholding of bad news, provides an explanation for asymmetric return volatility. They find that asymmetric volatility is more pronounced in the return series of individual firms that are more likely to disclose strategically as measured by their litigation risk incentives. Patterns in return volatility in market indices are also consistent with strategic disclosure as an explanation. As earnings management represents strategic decisions in mandatory reporting, different from strategic disclosure with verifiable reports, I do not present their findings as direct empirical evidence for this model. However, their paper suggests that managerial reporting decisions can matter in generating the observed patterns in stock returns, in line with the prediction of the current model.

sociation between idiosyncratic return volatility and financial reporting quality. The current model replicates the positive relationship between the likelihood of earnings management and the volatility of individual returns, and contributes to the theoretical explanations of the data.

In this paper, the contracting system in a principal-agent model with managerial reporting and moral hazard is first examined as a point of departure. This principal-agent model is developed and analyzed in greater detail in Sun (2008). The purpose of this step is to provide the underlying economic motive for earnings management in the model, to understand how motives to induce managerial effort and to motivate truthful reports differentially affect the optimal contract, and to identify how earnings management decision varies with actual economic performance. This principal-agent model lays out a micro-foundation for asset pricing in that it generates a set of earnings reports that may or may not be systematically biased. This model of managerial reporting under moral hazard is built on Dye (1988). The message space is limited to a single-dimensional signal while the privately informed agent receives two dimensions of private information; therefore the Revelation Principle is not applicable.[7]

In order to highlight the role that earnings management plays in price formulation, the principal-agent model with financial reporting choice is embcllded into an otherwise standard Lucas asset-pricing model. Iln particular, byswitching o and off the measure for earnings management in the model, I maintain the focus on earnings management and make the comparison with the standard asset-pricing model transparent. This modeling approach is related to Shorish and Spear (2005), where the owncr of the firm hires a manager to maximizc the firm's value, and there is asymmetric information about the manager's effort level between the owner and the manager. Along this line of agency-based asset pricing, Gorton and

[7] A recent paper by Crocker and Slemrod (2007) considers an alternative environment where the Revelation Principle can be applied. In solving the model, they assume a monotonically increasing reporting function; actual earnings can therefore be recovered by inverting the reporting function. In their setting, the principal knows the exact amount of actual earnings as a function of the report, while in the current model the principal faces uncertainty over whether earnings management occurs. The manager possesses a second dimension of private information in this model, and hence the reporting function is no longer invertible. As a model that constructs an explanation for earnings management, the current contract work can be viewed as complementary to theirs. As a microeconomic foundation for the investigation into asset pricing with earnings management, their model would generate prices that are fully revealing in the equilibrium; whereas the investors in this model try to infer the true outcomes through Bayesian learning, but cannot perfectly see through earnings management.

He (2006) show that when compensation depends on the firm's market performance, stock prices are set to induce the optimal effort level. In contrast with these papers, the current paper focuses on earnings management incentive in the contractual relationship and price formulation by assuming additional asymmetric information regarding output realizations.

This analysis also relates to the literature on asset pricing under asymmetric information, such as Detemple (1986), Wang (1993), and Cecchetti et al. (2000). In particular, Wang (1993) presents a dynamic asset-pricing model in which the investors can be either informed or uninformed: the informed investors know the future dividend growth rate; the uninformed investors do not. He finds that the existence of uninformed investors can lead to risk premia much higher than those under symmetric and perfect information. Distinguished from previous studies that examine the impact of information asymmetry and heterogeneous beliefs among investors, the study reported in this paper analyzes information asymmetry between corporate executives and outside investors as a whole.

There have not been many theoretical studies that examine the economic impact of earnings management. Fisher and Verrecchia (2000) is an early and notable exception. They show that more bias in the report reduces the correlation between share price and reported earnings, and study how the cost to the manager of biasing the report and the market's uncertainty about the manager's objective affect the slope and the intercept term in a regression of market price on the earnings report. Subsequently, Guttman et al. (2006) use a signaling model similar to Fischer and Verrecchia (2000) to explain the discontinuity observed in the distribution of reports. While these papers do not model the contractual relationship between shareholders and the manager, Kwon and Yeo (2007) consider a single-period model where the principal takes into account how compensation affects productive effort and market expectations when designing the optimal contract. In their paper, a rational market can simply recalibrate or discount the reported performance when the manager overstates earnings, and correctly guess the true performance. They show that such rational market discounting leads to less productive effort by the manager and less performance pay by the principal. In contrast with the studies presented in these papers, the current study considers stock returns under earnings management in a dynamic setting, with a central focus on the return properties beyond the first moment. This study further provides a quantitative evaluation of the model.

Existing studies have analyzed earnings management behavior and stylized financial facts in isolation, and a systematic investigation into the link between earnings management and financial anomalies has not yet been undertaken. By incorporating earnings management into an otherwise standard asset-pricing model, this paper presents a mechanism through which corporate misconduct may lead to a set of stylized financial facts. This paper suggests that there may be a unifying cause for these empirical regularities. In addition, the calibration results indicate that earnings management can be quantitatively important in explaining dynamic return patterns. This quantitative analysis suggests that earnings management by individual firms may not only generate patterns in their own stock returns, but also be powerful enough to create market-wide patterns.

The remainder of this paper proceeds as follows. Section 2 lays out the setup of the model. Section 3 discusses the general results, and presents the properties of simulated returns from the model. As one step toward calibration, Section 4 extends the model to continuous earnings. Section 5 presents a quantitative evaluation of the model. Section 6 checks the robustness of the model dynamics by adopting an alternative calibration strategy and incorporating stochastic investigation. Section 7 contains concluding remarks.

2 Model

The core of this paper is based on a Lucas asset-pricing model in which the investors hire a manager to operate the firm and report earnings. The investors design a contract that controls the manager's effort decision and reporting choice. In every period, the principal (investors) offers a newly hired manager a single-period contract. Earnings y are stochastic and take two possible values, $y \in \{l, h\}$, where $l < h$. The firm's production is associated with a simple Markov process:

$$\Pr(y_{t+1} = j | y_t = i) = \pi_{ij}, \qquad \forall i \in \{l, h\}, \quad \forall j \in \{l, h\}$$

The manager makes earnings announcements, and the reported earnings $R(y)$ are then paid out as dividends to the investors.[8] For simplicity, I assume that the manager finances the discrepancy in the report from a market ou

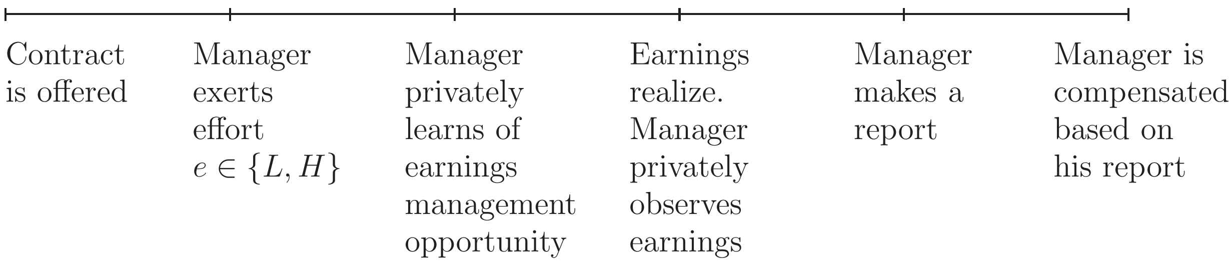

Figure 2: Timeline of contracting within each period

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The contractual environment follows Sun (2008). A risk-neutral principal (investors) hires a risk-averse agent (manager) for one period. Figure 2 details the timeline of the contracting arrangement between the principal and the manager. In the beginning of each period, the manager accepts the take-it-or-leave-it contract offered by the principal for one period. Earnings are stochastic and influenced by the manager's effort. The unobserved effort level of the manager, e, can take two values, low (L) and high (H). The manager incurs disuility from exerting effort, denoted by the cost function $a(e)$. In particular, high effort is associated with a cost of $a(H) = c$, and low effort involves no cost: $a(L) = 0$. Earnings take two possible values, represented by $y \in \{l, h\}$, where $l < h$. Let p_e be the probability that the earnings are equal to h when the effort is e, with $p_H > p_L$. After exerting effort, the manager privately learns whether he has the opportunity to manage earnings. With probability x, the manager has discretion over how much earnings to report. With probability $(1 - x)$, the manager is prohibited from manipulating earnings. Thus, in an economy where there are a large number of such investors and managers, x represents the percentage of managers able to manipulate earnings.[10] Then the manager privately observes the earnings, and makes an earnings announcement.[11]

[10] This paper considers a representative economy without firm heterogeneity.

[11] Here, whether the manager has the opportunity of managing earnings is assumed to be a random event, and the outcome is the manager's private information. Generally Accepted Accounting Principles (GAAP) provide guidelines on how to record and summarize each type of economic transaction, and hence define the accounting latitude available to senior management in financial reporting. In practice, certain economic activities, those where there is no hard-and-fast rule for which accounting method to use, lead to more discretion than others. In any particular period, economic transactions of this type may or may not take place. By virtue of being closer to the operations process, only the manager knows the extent of these

If the manager produces an inaccurate report, the manager incurs a personal cost, denoted by $\phi(\cdot)$. ϕ is a function of the discrepancy between true earnings and reported earnings. When the manager reports honestly, he incurs no cost: $\phi(0) = 0$.[12] When the manager overstates earnings, there is a positive cost $\phi(h - l) = \psi > 0$. Earnings management occurs in the model when the reported earnings differ from true earnings. More specifically, earnings management emerges in this environment if the manager announces that high earnings (h) have been achieved when the actual realization of earnings is low (l).

As the contract must be designed based on mutually observed variables, the manager's compensation can be based only on the earnings report. As long as the manager's reported earnings fall in the set $\{l, h\}$, the principal cannot directly detect whether the manager has misstated earnings. It is also assumed that the manager is essential to the operation of the firm, so the contract must be such that the manager (weakly) prefers to work for the principal regardless of whether the manager gains the opportunity to engage in earnings management.

To distinguish from high and low actual earnings, high and low reported earnings are denoted by $\tilde{h}$ and by $\tilde{l}$. The contract between the risk-neutral principal and the risk-averse agent includes a set of wages contingent on the reports, which can be alternatively characterized as a set of contingent utilities. The manager's utility level corresponding to compensation level w_i, $i \in \{\tilde{l}, \tilde{h}\}$, is denoted as $U(w_i) = u_i$, where $U(\cdot)$ is a strictly increasing and strictly concave utility function. Let $U^{-1}(\cdot) = V(\cdot)$. Then $V(u_i)$ is the cost to the principal of providing the agent with utility u_i. Because $U(\cdot)$ is a strictly increasing and strictly concave function, $V(\cdot)$ is a strictly increasing and strictly convex function.

The model presented in this section places restrictions on the manager's ability to communicate the truth. In addition to the unobserved effort level, the manager observes two dimensions of information, the value of actual earnings and the realization of misstatement opportunity. However, the manager is permitted to communicate only a one-dimensional

activities and hence the degree of reporting latitude available.

[12] There are two frictions in the model that restrain earnings management: earnings management opportunity that realizes with probability x and the cost involved in misstating earnings ϕ. This model can be also considered with only one friction: the cost of manipulation with a simple stochastic structure. The manipulation cost now in the model follows a binary distribution with two possible realizations ∞ and ψ.

signal, which is an earnings announcement. Communication is restricted in that the manager cannot fully communicate the full dimensionality of his information, and hence the Revelation Principle is not applicable.

In this environment, the contract must not only induce effort but also control for the manager's reporting incentive. This study assumes that the difference in the earnings is large enough that the principal always wants to implement high effort. The objective of the manager is to maximize utility by choosing a level of effort and a reporting strategy represented by $R(y)$, subject to the contract offered. When the manager has no discretion, we denote the report by $\bar{R}(h)$. By assumption, $\bar{R}(h) = \tilde{h}, \bar{R}(l) = \tilde{l}$. The manager's utility is of the form $U_m(e, R(y)) = xE[u_{R(y)} - \phi(R(y) - y) - a(e)] + (1 - x)E[u_{\bar{R}(y)} - a(e)]$. The first term is the manager's expected utility if the manager has discretion over reporting. The second term is the manager's expected utility if the manager does not have reporting discretion. The principal chooses the utility values u_i, $i \in \{\tilde{l}, \tilde{h}\}$, and recommended reporting choice $R(y)$ for each realization of earnings that minimize the expected cost of inducing effort.[13]

Formally, the optimal contract solves

$$\begin{aligned} \min_{u_{\tilde{h}}, u_{\tilde{l}}, R(h), R(l)} \quad & E[V(u)|H] \\ & = x[p_H V(u_{R(h)}) + (1 - p_H)V(u_{R(l)})] + (1 - x)[p_H V(u_{\tilde{h}}) + (1 - p_H)V(u_{\tilde{l}})] \end{aligned}$$

subject to

$$H = \underset{e \in \{L,H\}}{\arg\max}\, xE[u_{R(y)} - \phi(R(y) - y) - a(e)] + (1 - x)E[u_{\bar{R}(y)} - a(e)], \qquad \forall y \in \{l, h\}. \quad (1)$$

$$E[u|H] = xE[u_{R(y)} - \phi(R(y) - y) - a(e)|H] + (1 - x)E[u_{\bar{R}(y)} - a(e)|H] \geq \bar{U}. \quad (2)$$

The objective function is the expected cost for the principal to motivate high effort. The first term is the cost of implementing high effort when the manager has an opportunity to manage earnings, and the second term is the cost if the manager does not have the opportunity. The first constraint is the incentive constraint for the manager's effort choice — here, it is assumed that the principal wants to induce high effort. The second is the participation constraint, where $\bar{U}$ is the manager's outside option. In addition to these constraints, when

[13]As in the standard principal-agent model, the principal is the residual claimant, and hence entitled to receive the firm's earnings. The one-step departure from the standard model here is that the principal in this model does not observe the true earnings when the principal has to compensate the manager.

the manager has an opportunity to misstate earnings, the principal faces another constraint. As the reporting decision has been necessarily delegated to the manager, the "recommended reporting strategy" has to be voluntarily followed by the manager:

$$R(y) = \arg\max_{r \in \{\tilde{l}, \tilde{h}\}} u_r - \phi(r - y) \qquad \forall y \in \{l, h\}. \tag{3}$$

The optimal contract includes a set of utility promises $\{u_{\tilde{h}}, u_{\tilde{l}}\}$ and the recommended action $\{e^*, R(y)\}$. Following the convention, it is assumed that the principal wants to induce high effort, so $e^* = H$. The manager may take the following four possible reporting strategies:

Q ɛ **1** Report truthfully, that is, $\{R(h) = \tilde{h}, R(l) = \tilde{l}\}$.

Q ɛ **2** Report high earnings no matter which level of earnings is realized, that is, $\{R(h) = \tilde{h}, R(l) = \tilde{h}\}$.

Q ɛ **3** Report low earnings no matter which level of earnings is realized, that is, $\{R(h) = \tilde{l}, R(l) = \tilde{l}\}$.

Q ɛ **4** Report high earnings if low earnings are realized and report low earnings if high earnings are realized, that is, $\{R(h) = \tilde{l}, R(l) = \tilde{h}\}$.

It is straightforward to see that strategy 3 cannot be achieved without sacrificing effort, and strategy 4 cannot be implemented. The contracting problem is solved by characterizing the optimal payment schedule that implements high effort and each of strategy 1 and strategy 2, and then calculating the cost the principal incurs. The recommended reporting choice is the strategy that enables the principal to motivate high effort at the least cost, and the set of utility promises associated with the recommended reporting choice is the compensation schedule in the optimal contract. Below we will see that in some situations it is impossible to satisfy (1) and (3) simultaneously with a truthful report. In such a case, the principal has to endure a falsified report if the principal wants to implement high effort.

Figure 3 summarizes the main results. The optimal contract is described as the curve ABC, which depicts how the wedge between promised utilities assigned to reports of high and low earnings varies with different values of manipulation cost ψ. Below I restate the relevant results shown in Sun (2008).

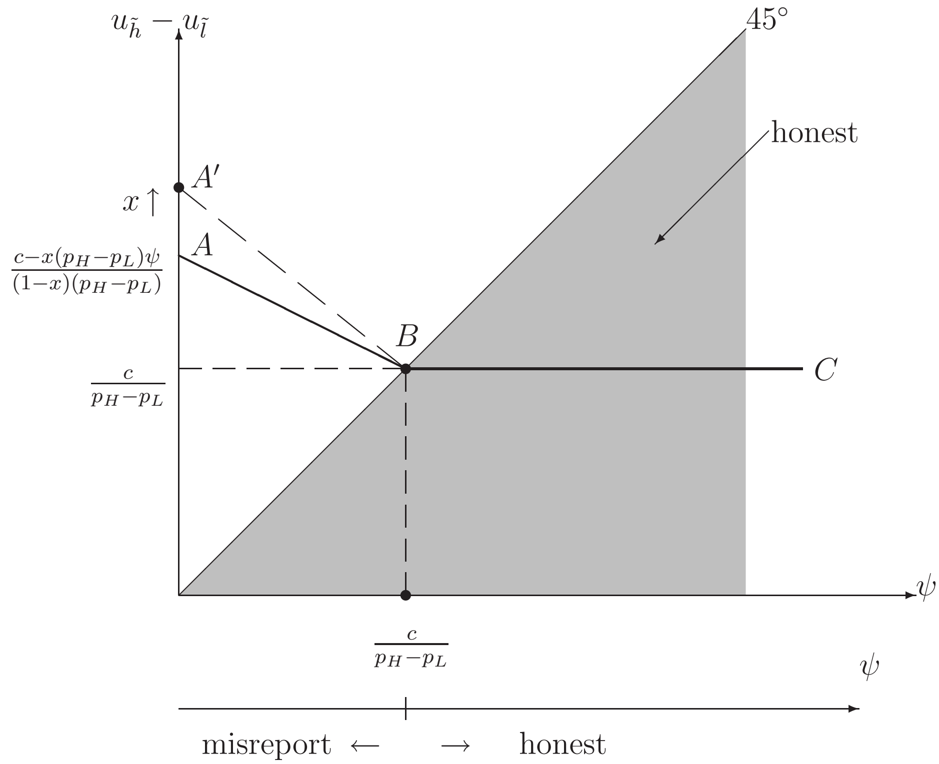

Figure 3: Main results

Lemma 1 *If $\psi < c/(p_H - p_L)$, truthful reporting is not implementable.*

Proof: Incentive compatibility constraint on truthful reporting (3) is

$$\begin{aligned} u_{\tilde{h}} - u_{\tilde{l}} &\leq \psi, \quad \text{if low earnings are realized,} \\ u_{\tilde{h}} - u_{\tilde{l}} &\geq 0, \quad \text{if high earnings are realized.} \end{aligned}$$

Because the principal cannot observe the realized true earnings, both have to be satisfied. Thus,

$$0 \leq u_{\tilde{h}} - u_{\tilde{l}} \leq \psi \tag{4}$$

has to hold. Incentive compatibility constraint on exerting high effort (1) is

$$p_H(u_{\tilde{h}} - c) + (1 - p_H)(u_{\tilde{l}} - c) \geq p_L u_{\tilde{h}} + (1 - p_L)u_{\tilde{l}},$$

which implies

$$u_{\tilde{h}} - u_{\tilde{l}} \geq \frac{c}{p_H - p_L}. \tag{5}$$

If $\psi < c/(p_H - p_L)$, incentive constraints on reporting choice (4) and effort decision (5) can not be both satisfied, therefore truthful reporting is not feasible. □

In Figure 3, the inequality (4) is represented by the shaded area below the 45° line. The inequality (5) is represented by the area above the horizontal line at $c/(p_H - p_L)$. For $\psi < c/(p_H - p_L)$, we can see that these areas do not overlap. If the cost of manipulating earnings is relatively small compared with the cost of exerting effort, it is infeasible to implement truthful reporting. When the principal attempts to control the manager's effort and reporting incentives, a sharp conflict arises between the desire of the principal to implement high effort, which requires substantial rewards on reports of high earnings, and the wish to motivate truthful reporting, which demands that the utility differential be small. The conflict makes it impossible to motivate the desired level of effort and truthful reporting at the same time, and the manager will always falsify the report when the chance presents itself. Earnings management emerges under the optimal contract.

Lemma 2 *If $\psi < c/(p_H - p_L)$ holds, the optimal contract satisfies*

$$u_{\tilde{h}} - u_{\tilde{l}} = \frac{c - x(p_H - p_L)\psi}{(1 - x)(p_H - p_L)}. \tag{6}$$

Proof: If $\psi < c/(p_H - p_L)$, from Lemma 1, truth-telling is not implementable. The only implementable reporting strategy is $\{R(h) = \tilde{h}, R(l) = \tilde{h}\}$. The incentive compatibility constraint on reporting choice (3) becomes

$$\begin{aligned} u_{\tilde{h}} - u_{\tilde{l}} &\geq \psi. \quad \text{if low earnings are realized} \\ u_{\tilde{h}} - u_{\tilde{l}} &\geq 0. \quad \text{if high earnings are achieved} \end{aligned}$$

Combining these two, we get

$$u_{\tilde{h}} - u_{\tilde{l}} \geq \psi \tag{7}$$

The incentive compatibility constraint on effort decision (1) in this case becomes

$$\begin{aligned} &x[p_H(u_{\tilde{h}} - c) + (1 - p_H)(u_{\tilde{h}} - \psi - c)] + (1 - x)[p_H(u_{\tilde{h}} - c) + (1 - p_H)(u_{\tilde{l}} - c)] \\ &\geq x[p_L u_{\tilde{h}} + (1 - p_L)(u_{\tilde{h}} - \psi)] + (1 - x)[p_L u_{\tilde{h}} + (1 - p_L)u_{\tilde{l}}], \end{aligned}$$

which can be simplified as

$$u_{\tilde{h}} - u_{\tilde{l}} \geq \frac{c - x(p_H - p_L)\psi}{(1-x)(p_H - p_L)}. \tag{8}$$

It must be binding in the optimal contract, and then the incentive compatibility constraint on reporting choice (7) is automatically satisfied. Suppose that the incentive constraint on effort decision (8) is not binding under the solution of the minimization problem. Then a small reduction in $u_{\tilde{h}}$ and an increase in $u_{\tilde{l}}$ that just keep the participation constraint (2) satisfied will still satisfy the incentive constraint (1). This change will reduce the value of the objective function. This contradicts to the supposition of the minimization. Hence, the incentive compatibility constraint on effort decision (8) is always binding. □

In Figure 3, the equation (6) is depicted by the line AB. As the principal designs the contract to control for effort choice and reporting behavior, the wedge between utilities assigned to high and low reports crucially depends on the cost of misstating earnings, the cost of making high effort, and the likelihood of having an opportunity to manipulate earnings. Relevant comparative statics are illustrated later in this section.

L 3 *If $\psi \geq c/(p_H - p_L)$, truthful reporting is the optimal solution.*

P: There are two possible reporting strategies the principal can implement: One strategy is reporting truthfully, that is, $\{R(h) = \tilde{h}, R(l) = \tilde{l}\}$, and the other choice is to report honestly if high earnings are realized and overstate earnings when low earnings are realized, that is, $\{R(h) = \tilde{h}, R(l) = \tilde{h}\}$.

If $\psi \geq c/(p_H - p_L)$ and the principal implements truthful reporting, the incentive compatibility constraint on effort decision (5) is binding, and hence the truthful-reporting constraint (4) is automatically satisfied. Suppose that the incentive compatibility constraint on effort decision (5) is not binding. Then a small reduction in $u_{\tilde{h}}$ and an increase in $u_{\tilde{l}}$ that just keep the participation constraint (2) satisfied will still satisfy the incentive constraint (5). This change will reduce the value of the objective function, resulting in a contradiction.

$u_{\tilde{h}}$ and $u_{\tilde{l}}$ can then be solved as follows:

$$\begin{aligned} u_{\tilde{h}} &= \bar{U} + c + \frac{c(1-p_L)}{p_H - p_L}, \\ u_{\tilde{l}} &= \bar{U} + c - \frac{cp_L}{p_H - p_L}. \end{aligned}$$

If $\psi \geq c/(p_H - p_L)$ and the principal implements the alternative strategy, $\{R(h) = \tilde{h}, R(l) = \tilde{h}\}$. As shown in Lemma 2, the incentive compatibility constraint on effort decision (8) must be binding in the optimal contract, and then the incentive compatibility constraint on reporting choice (7) is automatically satisfied. $u_{\tilde{h}}$ and $u_{\tilde{l}}$ can be solved as follows:

$$
\begin{aligned}
u_{\tilde{h}} &= \bar{U} + c + (1 - p_H)\psi, \\
u_{\tilde{l}} &= \bar{U} + c - p_H\psi.
\end{aligned}
$$

Compared to the case with the alternative reporting strategy, implementing truthful-reporting strategy requires a lower $u_{\tilde{h}}$ and a higher $u_{\tilde{l}}$, and hence makes the utility promises more equalized. Given the convex cost of providing utilities, it incurs a lower cost in inducing effort with truth-telling strategy. Truthful reporting is the optimal solution in this case. □

If the cost of misstating earnings is large compared with the cost of exerting effort, it is relatively easy to motivate truthful earnings reports. When truth-telling strategy is feasible, it is always in the principal's best interest to achieve truthful reporting. The principal avoids earnings management whenever feasible, because the principal eventually bears the cost of misreporting. Although manipulation is personally costly to the manager, because the principal must design a compensation contract that meets the manager's participation constraint, the cost of manipulation undertaken by the manager is ultimately borne by the principal.

Proposition 1 *$u_{\tilde{h}} > u_{\tilde{l}}$ always holds.*

Proof: See the proof of Lemma 2 and Lemma 3. □

As in the standard contracting problem, reports of high earnings are associated with a larger compensation in order to motivate the preferable effort.

When does earnings management occur? The following proposition establishes necessary and sufficient conditions for earnings management to emerge under the optimal contract.

Proposition 2 *$\psi < c/(p_H - p_L)$ is the necessary and sufficient condition for earnings management to occur under the optimal contract.*

Proof: Straightforward from Lemma 1 and Lemma 3. □

Here, the optimal contract is fully characterized, and the condition for earnings management to take place is derived explicitly. As the investors have to control for the manager's effort decision and reporting strategy, there is a tension between inducing managerial effort and motivating truthful reporting. A relatively sensitive payment schedule generates an incentive to manage earnings, whereas a compensation schedule that is not responsive enough fails to motivate the desired level of effort. If the cost of manipulating earnings is relatively small compared to the cost of exerting effort, it is prohibitively difficult to implement truthful reporting while maintaining the manager's incentive to exert effort, and earnings management is sustainable as equilibrium behavior.

Proposition 3 *Suppose that $\psi < c/(p_H - p_L)$ holds. Then $u_{\tilde{h}} - u_{\tilde{l}}$ is decreasing in ψ.*

Proof: From Lemma 2,

$$u_{\tilde{h}} - u_{\tilde{l}} = \frac{c - x(p_H - p_L)\psi}{(1-x)(p_H - p_L)}. \tag{9}$$

It can easily be checked that the right-hand-side of (9) is decreasing in ψ. □

Suppose that, possibly due to a more stringent accounting rule or corporate governance policy, misstating earnings becomes more costly to the manager. Then, if the low outcome realizes and the manager has an opportunity to inflate earnings, the manager will propel earnings upward, but this overstatement of earnings is more costly. The manager has more incentive to avoid this situation, and this works as an additional incentive for the manager to work hard. Thus, the principal does not have to provide as much monetary incentive ($u_{\tilde{h}} - u_{\tilde{l}}$) to satisfy the incentive compatibility constraint (1).

Proposition 4 *Suppose that $\psi < c/(p_H - p_L)$ holds. Then $u_{\tilde{h}} - u_{\tilde{l}}$ is increasing in x.*

Proof: As in Proposition 3, (9) holds. (9) can be rewritten as follows.

$$u_{\tilde{h}} - u_{\tilde{l}} = \frac{c - (p_H - p_L)\psi}{(1-x)(p_H - p_L)} + \psi. \tag{10}$$

It can easily be checked that the right-hand-side (10) is increasing in x. □

In Figure 3, the line AB shifts to $A'B$ as x increases. Suppose that x becomes greater, suggesting that the manager is more likely to be able to manipulate earnings. The manager then enjoys a higher chance of being able to overstate earnings when low earnings are realized, which leads to less incentive to make high effort under any given compensation schemes. A larger reward for high earnings report is thus required to overcome the manager's motive to slack off, hoping to later bump up earnings. As a result, the executive compensation schedule becomes steeper.

As x represents the probability of the manager being dishonest, x also indicates the percentage of managers engaging in earnings management in the economy as a whole. Compared with an economy without opportunities to manage earnings ($x = 0$), in an economy where it is possible ($x > 0$) and not too costly to manipulate earnings ($\psi < c/(p_H - p_L)$), it is optimal for the shareholders to provide stronger monetary incentives to executives through the compensation packages they offer. When x, the index for the prevalence of earnings management, rises, the model predicts an executive compensation structure that is more responsive to performance.

It is worth pointing out that the principal in the model compensates the manager with utility promises contingent on earnings reports, and a larger utility differential does not necessarily translate into a larger wage difference. In this model, the incentive compatibility constraint on effort choice (1) determines the utility differential between high and low reports, and the participation constraint (2) pins down the exact levels of promised utilities:

$$u_{\tilde{h}} = \bar{U} + \frac{c(1-p_L)}{(p_H - p_L)}, \tag{11}$$

$$u_{\tilde{l}} = \bar{U} + \frac{c(1-p_L)}{(p_H - p_L)} - \frac{c - x(p_H - p_L)\psi}{(1-x)(p_H - p_L)}. \tag{12}$$

Because $u_{\tilde{h}}$ is independent of ψ and x, the change of $(u_{\tilde{h}} - u_{\tilde{l}})$ is solely due to the change of $u_{\tilde{l}}$. It is straightforward to map the utility wedge into the wage differential in this case.

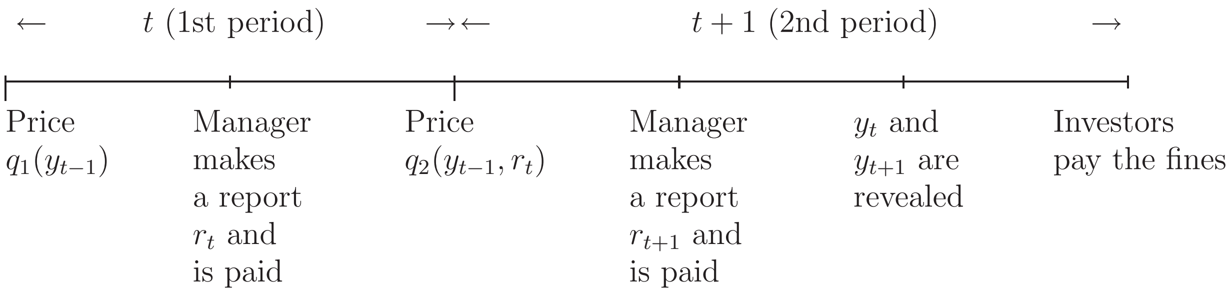

Figure 4: Model timeline

2.2 Ass[illegible] Q[illegible]

Now, this contract model is embedded into a dynamic model of asset pricing. It is assumed that the earnings process is persistent: the true earnings at time t, y_t, depends on y_{t-1} in addition to the manager's current effort. In particular, under the high effort by the manager (which is always the case in the equilibrium I consider), I assume that the true earnings follow a Markov process with transition probability $\pi_{yy'}$, where y is the earnings at time $t-1$ and y' is the earnings at time t. The asset price is determined as the present value of the dividends, which are the reported earnings net of compensation and financial charges for earnings management. Figure 4 chronicles the timeline of the model. It describes the timing of the events in two consecutive periods t and $t+1$, and this two-period auditing cycle repeats over time. Because the model is stationary, all the relevant past information is summarized in the previously revealed earnings and current reported earnings.

In the first period of the two-period auditing cycle (hereafter, period 1), the price of the firm $q_1(y_{t-1})$ is determined based on the revelation of the previous period's earnings y_{t-1}. Having the manager's earnings management incentive in mind, the investors form their expectations about future dividend income based on the revelation of the firm's previous earnings y_{t-1}. In the second period of each cycle (hereafter, period 2), given the earnings report in the first period r_t and the true outcome in the ending period of the last cycle y_{t-1}, the firm is priced as $q_2(y_{t-1}, r_t)$. After the manager reports the earnings and pays them out entirely to the investors, the investigation takes place. When the investigation is conducted, the true realization of earnings in each period of the cycle is revealed. The investors bear the financial punishment associated with any misstatement of earnings that occurs during the cycle. If a false report occurs in one of the two periods, an amount of penalties F_1 is charged. If false earnings reports occur in both periods, the investors must pay an amount of fines F_2, where $F_2 \geq 2F_1$.

I assume that the investors have linear utility, and maximize the sum of the expected dividends. Then the value of the firm can be formulated as follows. In the beginning of an auditing cycle, given the revelation of the true outcome in the end of the last cycle y_{t-1}, the price of the firm $q_1(y_{t-1})$ is given by the expected sum of the net dividends and asset price in the next period (the time subscript is dropped when the timing is clear):

$$\begin{aligned} q_1(h) =& \pi_{hh}[d_{\tilde{h}} + \beta q_2(h, \tilde{h})] + \pi_{hl} x[d_{\tilde{h}} + \beta q_2(h, \tilde{h})] \\ &+ \pi_{hl}(1-x)[d_{\tilde{l}} + \beta q_2(h, \tilde{l})], \end{aligned} \tag{13}$$

and

$$\begin{aligned} q_1(l) =& \pi_{lh}[d_{\tilde{h}} + \beta q_2(l, \tilde{h})] + \pi_{ll} x[d_{\tilde{h}} + \beta q_2(l, \tilde{h})] \\ &+ \pi_{ll}(1-x)[d_{\tilde{l}} + \beta q_2(l, \tilde{l})], \end{aligned} \tag{14}$$

where d_r is the net dividend income and β is the investors' discount factor. The net dividend income equals the reported earnings less the compensation, that is, $d_r = r - w(r)$, where $r \in \{\tilde{l}, \tilde{h}\}$.

Regardless of the revelation of y_{t-1} in period 1, the investors may encounter three possible states in period 2. The first term in (13) and (14) is the expected net dividend income if the manager sends an honest report of high earnings in the next period. The second term in (13) and (14) represents the case in which the actual realization of earnings is low, but the manager makes an overstatement of earnings. The third term in the prices is the case in which the manager truthfully reports low earnings.

Given the first-period report r_t and the previously revealed outcome y_{t-1}, the investors update their belief about the true state in period 1. If the first-period report is low, it is for certain an honest report. If the report sent by the manager is high, it may be an overstated report that leads to immediate penalties. The posterior belief of the first-period report being truthful is derived following Bayes' Rule. If the previously revealed outcome is high, the conditional probability of $y_t = h$, denoted by γ_1, is

$$\begin{aligned}\gamma_1 &= \Pr(y_t = h | r_t = \tilde{h}, y_{t-1} = h) \\ &= \frac{\Pr(r_t = \tilde{h} | y_t = h) \Pr(y_t = h | y_{t-1} = h)}{\Pr(r_t = \tilde{h} | y_{t-1} = h)} \\ &= \frac{\pi_{hh}}{\pi_{hh} + \pi_{hl} x},\end{aligned}$$

If the previously revealed outcome is low, the conditional probability of $y_t = h$, denoted by γ_2, is

$$\begin{aligned}\gamma_2 &= \Pr(y_t = h | r_t = \tilde{h}, y_{t-1} = l) \\ &= \frac{\Pr(r_t = \tilde{h} | y_t = h) \Pr(y_t = h | y_{t-1} = l)}{\Pr(r_t = \tilde{h} | y_{t-1} = l)} \\ &= \frac{\pi_{lh}}{\pi_{lh} + \pi_{ll} x}.\end{aligned}$$

The price of the firm $q_2(y_{t-1}, r_t)$ is determined using these posterior probabilities. There are two cases. First, if period 1's report is low, the investors know that the realization of earnings is low.

$$q_2(l, \tilde{l}) = q_2(h, \tilde{l}) = \pi_{lh} \left[d_{\tilde{h}} + \beta q_1(h)\right] + \pi_{ll} x \left[d_{\tilde{h}} - F_1 + \beta q_1(l)\right] + \pi_{ll} (1 - x) \left[d_{\tilde{h}} + \beta q_1(l)\right]. \tag{15}$$

Because actual earnings follow a Markov process, the most recent realization of earnings is the only useful information for predicting future earnings. The price in response to a low report (which implies a realization of low earnings) is thus independent of the previous revelation of earnings, equal the expected payoff over three possible states in the next period. The first term in (15) is the expected net dividend income if the manager sends an honest report of high earnings in the current period. The second term in (15) represents the case in which the manager makes an overstatement o truthful. Prices are determined as follows:

$$q_2(h,\tilde{h}) = \tag{16}$$

$$\gamma_1 \left\{ \pi_{hh} \left[d_{\tilde{h}} + \beta q_1 \left(h \right) \right] + \pi_{hl} x \left[d_{\tilde{h}} - F_1 + \beta q_1 \left(l \right) \right] + \pi_{hl} \left(1 - x \right) \left[d_{\tilde{l}} + \beta q_1 \left(l \right) \right] \right\}$$
$$+ \left(1 - \gamma_1 \right) \left\{ \pi_{lh} \left[d_{\tilde{h}} - F_1 + \beta q_1 \left(h \right) \right] + \pi_{ll} x \left[d_{\tilde{h}} - F_2 + \beta q_1 \left(l \right) \right] + \pi_{ll} \left(1 - x \right) \left[d_{\tilde{l}} - F_1 + \beta q_1 \left(l \right) \right] \right\},$$

$$q_2(l,\tilde{h}) = \tag{17}$$

$$\gamma_2 \left\{ \pi_{hh} \left[d_{\tilde{h}} + \beta q_1 \left(h \right) \right] + \pi_{hl} x \left[d_{\tilde{h}} - F_1 + \beta q_1 \left(l \right) \right] + \pi_{hl} \left(1 - x \right) \left[d_{\tilde{l}} + \beta q_1 \left(l \right) \right] \right\}$$
$$+ \left(1 - \gamma_2 \right) \left\{ \pi_{lh} \left[d_{\tilde{h}} - F_1 + \beta q_1 \left(h \right) \right] + \pi_{ll} x \left[d_{\tilde{h}} - F_2 + \beta q_1 \left(l \right) \right] + \pi_{ll} \left(1 - x \right) \left[d_{\tilde{l}} - F_1 + \beta q_1 \left(l \right) \right] \right\}.$$

The first term in (16) and (17) corresponds to the case where the first-period report is honest. In this case, there are three possible situations in the next period. In particular, if the realization of the second-period earnings is low and the manager has an opportunity to inflate earnings, the manager will report high. An amount of monetary penalties F_1 will be charged, and thus subtracted in the pricing equation. The second term in (16) and (17) represents the case in which the first-period report is false. There are again three possible states in the second period. The investors pay an amount of fines F_1 if the manager truthfully presents earnings in period 2 and an amenalties

2.3 Ç Q

The price differential between $q_1(h)$ and $q_1(l)$ measures how sensitive the firm's price $q_1(y_{t-1})$ is in response to the investigation results y_{t-1}. How does $q_1(h) - q_1(l)$ change as the opportunity of earnings management, x, changes? To examine this, let us first ignore that the wage of the manager actually changes with x. It can be shown that as long as the firm's stochastic production process is persistent, that is, $\pi_{hh} > \pi_{lh}$, the price becomes more responsive to investigation results as x increases. Under the condition that $\beta F_1 > (d_{\tilde{h}} - d_{\tilde{l}})$, both $q_1(h)$ and $q_1(l)$ fall as x escalates. However, $q_1(l)$ diminishes faster than $q_1(h)$, because a low previous output implies that future outputs tend to be low as well, imposing greater exposure to earnings restatement risk.

The analysis above does not consider that wages and thus net dividend income change with x. However, the same qualitative result holds even if the change in the compensation is taken into account. The optimal contract in this environment is characterized by (11) and (12). It can be seen that the compensation for the report of high earnings is independent of x, and the compensation for low earnings reports decreases as x expands. Therefore, as x becomes greater, the net dividend income from a report of high earnings, that is, $d_{\tilde{h}} = \tilde{h} - w(\tilde{h})$, remains the same, whereas the net dividend from a low earnings report, $d_{\tilde{l}} = \tilde{l} - w(\tilde{l})$, increases, resulting in a smaller dividend differential between high and low reports. Assuming that the monetary penalties F_1 and F_2 do not vary with x, as the financial gain from earnings management, represented by $d_{\tilde{h}} - d_{\tilde{l}}$, diminishes, earnings management becomes more financially costly to the investors. The prices thus drop more as x rises. The change in the compensation schedule in response to the change of x internalizes the financial gain from earnings management, and it reinforces the amplification of the price differential and hence the price volatility.

Keeping the revelation of previous earnings constant, the price wedge in response to different reports in the ending period of one cycle, as measured by $q_2(h, \tilde{h}) - q_2(h, \tilde{l})$, does not necessarily have a monotonic relationship with x. To see this in a relatively straightforward manner, let us first ignore the effect of x on the manager's wages. $q_2(h, \tilde{h})$ is decreasing in x because of two forces that reinforce each other. First, as x rises, it is more likely to have false reports in future. These falsified reports lead to the investors' financial losses. Second, it is also more likely that the previous report r_t is a false report, resulting in fines waiting to be paid. Because $\tilde{l}$ in $q_2(h, \tilde{l})$ is surely an honest report, the second force is absent. However, we do not necessarily obtain a smaller gap between $q_2(h, \tilde{h})$ and $q_2(h, \tilde{l})$ as x increases. Because of the high persistence in the earnings process, the first force works stronger for $q_2(h, \tilde{l})$ than for $q_2(h, \tilde{h})$. The impact of changes in x on the price volatility remains ambiguous in this case.

There are additional effects to consider if we take into account the impact of x on compensation schedule. Recall that the compens

Parameter	Value
h	50
l	0
π_{hh}	0.8
π_{ll}	0.8
β	0.95
F_1	$1.2(h-l)/\beta$
F_2	$2F_1$

Table 1: Parameter values in the numerical example with binary earnings

is measured as an equally weighted average of the return volatility in each period of one auditing cycle. In the revelation stage, it is straightforward to show that volatility rises with x. When earnings management opportunities become more likely, more frequent earnings restatements generate greater fluctuations in the returns and thus higher volatility. Earnings management may dampen return volatility in the reporting periods, because there is less variation in the reports. In addition, the prices in response to high reports are discounted to reflect possible earnings management, leading the price range to shrink. However, earnings management amplifies the movement of returns in the revelation stage significantly. As long as F_1 is not too small, the amplification effect of earnings restatement risk in the revelation stage is dominant, and hence average return volatility increases with x.

Analogously, in order to compare the conditional volatility difference in response to earnings revelations, I use the difference between the equally weighted average of the return volatility in one cycle following a revelation of high earnings and that following a revelation of low earnings. If $x = 0$, it is straightforward to show that the difference is zero. With a positive value of x, earnings restatement risk increases the volatility difference in the revelation stage, because low earnings generate financial incentives for the manager to overstate earnings while high earnings do not. As long as F_1 is large enough, the asymmetry in return volatility is present when earnings management is possible.

3 Results

In this section, I solve the model numerically, and present the results from model simulations. Table 1 shows the parameter values. The primary purpose in this section is to illustrate that earnings management can generate a number of stylized financial facts. The quantitative results will be presented in Section 5.[16]

3.1 [illegible]

For the illustrative purpose, I use $x = 0$ and $x = 0.1$ as an example to demonstrate the impact of earnings management throughout this section. The simulated return sequence from the model captures the stylized facts of conditional volatility: first, conditional volatility exhibits persistence; second, stock returns are negatively correlated with the volatility of subsequent returns.

The EGARCH (1,1) model of the return series is estimated using Maximum Likelihood method with 10,000 artificially generated observations. The EGARCH (1,1) model used is $\log \sigma_t^2 = K + G \log \sigma_{t-1}^2 + A[|\epsilon_{t-1}|/\sigma_{t-1} - E\{|\epsilon_{t-1}|/\sigma_{t-1}\}] + L[\epsilon_{t-1}/\sigma_{t-1}]$, where E is the expectation operator, ϵ_t is the innovation, and σ_t is the conditional variance of the innovation. The G term captures volatility clustering (that is, persistence of volatility). A positive value

[16] It is worth noting that the asset pricing model is consistent with the contract model in the sense that it is optimal for the investors to implement high effort when designing executive compensation, although earnings management leads to monetary penalties imposed on the investors. Recall that in the contract model with two-earnings-level specification, the principal always wants to induce high effort. In the following analysis, wage values are assumed to be negligibly small relative to firms' earnings. In a standard principal-agent model without earnings management, high effort is desirable as long as high earnings are different enough from low earnings. With the possibility of earnings management and restatement announcements, it is still beneficial for the principal to induce high effort if the value of high effort outweighs the possible monetary loss associated with earnings management. That is,

$$[p_H h + (1 - p_H)l] - [p_L h + (1 - p_L)l] > xF_1 \tag{18}$$

And recall that for earnings management to exert influence on stock returns, the discounted monetary

x=0	Coefficient	Std.Error	t-statistic
K	-5.0000	0.4153	-12.0387
G	-0.0001	0.6829	0.0001
A	0.0000	0.0087	0.0000
L	0.0009	0.0092	0.1049

x=0.1	Coefficient	Std.Error	t-statistic
K	-1.8621	0.3136	-5.9380
G	0.5999	0.0663	9.0545
A	0.0407	0.0058	6.9856
L	-0.1125	0.0278	-4.0553

Table 2: EGARCH(1,1) estimation results

Variance equation: $\log\sigma_t^2 = K + G\log\sigma_{t-1}^2 + A[|\epsilon_{t-1}|/\sigma_{t-1} - E\{|\epsilon_{t-1}|/\sigma_{t-1}\}] + L[\epsilon_{t-1}/\sigma_{t-1}]$

of the A term in the equation implies that a deviation of the standardized innovation from its expected value causes the variance to be larger than otherwise. The L coefficient allows this effect to be asymmetric.[17]

Table 2 presents the results. The upper panel presents the case without earnings management, that is, $x = 0$. In this case, there is no GARCH or ARCH effect present in the simulated return data. As x becomes positive, return volatility becomes serially correlated. Before estimation, the Lagrange Multiplier (LM) test is applied to the return data, and the LM test strongly rejects the i.i.d. residual hypothesis at the 95% confidence level. The coefficients of the EGARCH (1,1) model are all statistically significant beyond the 95% confidence level. In addition, the conditional variance process is strongly persistent (with G coefficient = 0.60). The negative value of the coefficient L gives evidence of asymmetry in the model return behavior — negative surprises increase volatility more than positive surprises.

The persistence and asymmetry in the conditional volatility of stock returns in the model is generated by earnings management incentive together with a persistent earnings process. When earnings are revealed to be low, the persistence in the earnings-generating process implies that earnings tend to stay low for a while, so earnings management is likely to occur

[17]If $L = 0$, then a positive surprise ($\epsilon_{t-1} > 0$) has the same effect on volatility as a negative surprise of the same magnitude. If $-1 < L < 0$, a positive surprise increases volatility less than a negative surprise. If $L < -1$, a positive surprise actually reduces volatility while a negative surprise increases volatility. For further reference, see Hamilton (1994, p. 668).

x	Standard Deviation
0	0.0954
0.1	0.1015
0.2	0.1086

Table 3: Volatility of the model returns

in the current and future periods. A higher frequency of occurrence of earnings management increases future return volatility. If the previous earnings are revealed to be high, the current and future earnings are likely to remain high. Overstatement of earnings has little chance of occurring; thereby future returns are relatively stable in this case. As a result, the volatility of the return series is persistent, and returns are negatively correlated with the subsequent volatility.

Note that the core intuition does not hinge upon the two-period time structure of information disclosure. The mechanism that drives EGARCH property stays in effect when the model is extended to incorporate additional periods and stochastic investigation in Section 6.2. As long as restatements generate returns movements and there is a persistent component to earnings management because of the persistence in underlying profitabilities, the substance of the model dynamics remains.

3.2 R[illegible] Q[illegible]

Table 3 presents the volatility of returns in the simulated data. Monetary penalties charged during earnings restatement generate large swings in the return sequence and hence raise volatility. When earnings management and earnings restatement occur more frequently, returns become more volatile. Campbell et al. (2001) document that idiosyncratic stock return volatility increased considerably from 1962 to 1997 in the United States. Rajgopal and Venkatachalam (2007) report a strong association between idiosyncratic return volatility and financial reporting quality, as measured by both earnings quality and forecast dispersion, in both cross-sectional and time-series regressions. In line with the empirical findings, as x increases in the model, implying that the informativeness of earnings reports becomes weakened, the returns exhibit greater volatility.

4 E[illegible]

In this section,the model is extended to the case with a continuum of earnings. This model is used for the quantitative analysis in the next section. In the continuous case, I assume that earnings follow an AR(1) process: $y' = \rho y + k + \epsilon$, where $\rho < 1$, k is a constant, and ϵ is a white noise process with zero mean and standard deviation σ.

4.1 O[illegible]

Analogous to the binary model elaborated above, a risk-neutral principal (investors) hires a risk-averse agent (manager) for one period. Expending high effort incurs a utility cost, that is, c, to the manager, whereas low effort involves no cost. The manager's effort decision and an exogenous state realization together determine the firm's economic earnings, which is privately observed by the manager. The conditional distributions of earnings given high and low effort follow normal distributions: $f(y$ [illegible] subject to

$$H = \underset{e \in \{L,H\}}{\arg\max}\, xE\Big\{U\left[w\left(R(y)\right)\right] - \phi\left(R(y) - y\right) - a(e)\Big\} + (1-x)E\Big\{U\left[w(y)\right] - a(e)\Big\}, \quad y_e \sim N(\mu_e, \sigma_e). \tag{20}$$

$$E[u|H] = xE\Big\{U\left[w\left(R(y)\right)\right] - \phi\left(R(y) - y\right) - a(e)|H\Big\} + (1-x)E\Big\{U\left[w(y)\right] - a(e)|H\Big\} \geq \bar{U}. \tag{21}$$

The objective function is the expected wage payment for the principal to motivate effort. The first term is the expected payment to implement effort when the manager has an opportunity to artificially inflate earnings, and the second term is the wage if the manager does not have such an opportunity. The principal designs a compensation contract that satisfies the incentive constraint on the effort decision (20) and the participation constraint (21). In addition to these constraints, when the manager has an opportunity to exaggerate earnings, the "recommended reporting strategy" has to be in the manager's best interest. This incentive constraint on the reporting strategy in the continuous case is:

$$R(y) = \underset{r \in \{y, y+a\}}{\arg\max}\, U\left[w(r)\right] - \phi(r - y) \qquad \forall y \sim N(\mu_H, \sigma_H). \tag{22}$$

More specifically,

$$R(y) = y \qquad \text{if } U\left[w(y+a)\right] - U\left[w(y)\right] < \psi, \tag{23}$$

$$R(y) = y + a \qquad \text{if } U\left[w(y+a)\right] - U\left[w(y)\right] \geq \psi. \tag{24}$$

The optimal wage schedule is numerically computed in Sun (2008), utilizing Simulated Annealing algorithm with Gauss Hermite quadrature. The Schumaker approximation is used to preserve the shape of wage functions in interpolation and extrapolation. In the numerical implementation, it is always the case that under the optimal contract, there exists a threshold level of earnings y^*, above which the manager does not find it worthwhile to manipulate earnings and truth-telling strategy is thus maintained. Below this threshold, the manager achieves personal gains from manipulation, and inflates earnings whenever possible. Thereafter, this paper focuses on this threshold-style of reporting behavior.

The intuition behind the existence of the threshold earnings that separates truthful reporting and earnings management is as follows. Given that the manager is risk averse, a wage function that is not too convex translates into a set of concave utility promises. As actual earnings expand, the manager faces a decreasing utility gain but a constant utility cost from overstating earnings. As a consequence, earnings management occurs when the realized earnings are relatively low, and a truthful reporting strategy is sustained if the actual earnings are high.

4.2 Asset Quotes

The pricing formulation is extended to the continuous case as follows.[18] Based on the revelation of the previous earnings, the price in period 1 is determined as the expected sum of the dividends and price in the next period:

$$\begin{aligned} q_1(y) = & \Pr[y' \geq y^*|y] E\left[(\rho y + k + \epsilon) + \beta q_2(y, \rho y + k + \epsilon)|y' \geq y^*\right] \\ & + \Pr[y' < y^*|y] x E\left[(\rho y + k + \epsilon + a) + \beta q_2(y, \rho y + k + \epsilon + a)|y' < y^*\right] \\ & + \Pr[y' < y^*|y](1-x) E\left[(\rho y + k + \epsilon) + \beta q_2(y, \rho y + k + \epsilon)|y' < y^*\right]. \end{aligned} \quad (25)$$

The first term in the pricing function represents the case when the actual earnings in the next period exceed the threshold level of earnings that elicits the truth, and therefore the manager reports honestly. The second term in (25) is the case when the next period's actual earnings fall below the threshold earnings, and the manager has an opportunity to manage earnings. The manager in this case overstates earnings. In particular, the next period's report is $r = \rho y + k + \epsilon + a$. The third term in (25) represents the situation in which the next period's earnings are below the threshold earnings, but the manager does not have the earnings management opportunity. In this case, the manager has to truthfully represent the earnings.

The price in period 2 is a function of the previously revealed earnings and the earnings report in period 1.

$$q_2(y, r) = p\Omega + (1-p)\tilde{\Omega},$$

[18]Again, the labor wage is assumed to be negligibly small compared with the firm's earnings, therefore compensation does not affect net dividends or asset prices.

where

$$\Omega \equiv \Pr[y'' \geq y^*|y' = r]E\left[(\rho r + k + \epsilon) + \beta q_1(\rho r + k + \epsilon)|y'' \geq y^*\right]$$
$$+ \Pr[y'' < y^*|y' = r]xE\left[(\rho r + k + \epsilon + a) - F_1 + \beta q_1(\rho r + k + \epsilon)|y'' < y^*\right]$$
$$+ \Pr[y'' < y^*|y' = r]\,(1 - x)\,E\left[(\rho r + k + \epsilon) + \beta q_1(\rho r + k + \epsilon)|y'' < y^*\right],$$

and

$$\tilde{\Omega} \equiv \Pr[y'' \geq y^*|y' = r - a]E\left[(\rho(r - a) + k + \epsilon) - F_1 + \beta q_1\left(\rho(r - a) + k + \epsilon\right)|y'' \geq y^*\right]$$
$$+ \Pr[y'' < y^*|y' = r - a]xE\left[(\rho(r - a) + k + \epsilon + a) - F_2 + \beta q_1\left(\rho(r - a) + k + \epsilon\right)|y'' < y^*\right]$$
$$+ \Pr[y'' < y^*|y' = r - a](1 - x)E\left[(\rho(r - a) + k + \epsilon) - F_1 + \beta q_1\left(\rho(r - a) + k + \epsilon\right)|y'' < y^*\right].$$

Here, Ω is the expected present value of the dividends when the first-period report is truthful, and $\tilde{\Omega}$ corresponds to the case where the first-period report is false. Similar to the pricing function in period 1, the first term in Ω and $\tilde{\Omega}$ represents the case when the second-period earnings are higher than the threshold earnings, and the reported earnings are truthful. In $\tilde{\Omega}$, F_1 is subtracted because investors must pay monetary penalties for the earnings management practice in period 1 of this auditing cycle. The second term in Ω and $\tilde{\Omega}$ represents the case when the actual earnings in period 2 are lower than the threshold earnings, and the manager has discretion to inflate earnings by a. In this case, the investors pay F_1 for the overstatement if the first-period report is honest (as in Ω) and F_2 if the first-period report is also falsified (as in $\tilde{\Omega}$). The third term is the case when the manager does not have any discretion over reporting, and has to truthfully report the earnings that fall below the threshold earnings. In $\tilde{\Omega}$, the deduction of F_1 is due to the earnings overstatement by the manager in period 1.

The posterior belief of having an accurate report in period 1, that is, $p = \Pr[y' = r|y]$, is derived following Bayes' Rule,

$$p = \begin{cases} 1 & \text{if } r \in [y^* + a, \infty), \\ \dfrac{f(r - k - \rho y)}{f(r - k - \rho y) + xf(r - a - k - \rho y)} & \text{if } r \in (y^*, y^* + a), \\ \dfrac{(1 - x)f(r - k - \rho y)}{(1 - x)f(r - k - \rho y) + xf(r - a - k - \rho y)} & \text{if } r \in (-\infty, y^*]. \end{cases} \tag{26}$$

Note that the compensation contract endogenously determines the threshold level y^* that elicits the truth. As actual earnings follow an AR(1) process, the implied conditional distributions of earnings given effort change over time, leading to changes of compensation contracts and hence threshold levels. In the simulation of prices and returns, the endogeneity of y^* requires calculations of the optimal contract for each possible earnings distribution implied by previous earnings. Sun (2008) specifies the parameterization of the principal-agent model such that the threshold level equals the conditional mean of actual earnings given high effort. The following proposition states the conditions under which the wage schedule shifts in a parallel manner when the earnings distribution moves. More specifically, the optimal contract and the underlying earnings distribution move together in the same direction by an equal amount. Therefore, the threshold level is always equal to the mean of earnings given high effort, even when the mean level itself varies over time.[19]

Proposition 5 *Suppose that the values of the parameters $(a, \psi, c, \bar{U}, \sigma_H, \sigma_L)$ are fixed, and $f(y|e = H)$ and $f(y|e = L)$ shift in a parallel manner by δ, keeping $(\mu_H - \mu_L)$ fixed. Then a parallel shift of the wage function $w(r)$ by δ is a solution to the principal's problem, and therefore the threshold level y^* will shift by δ as well.*

Proof: See Appendix.

Below, I restrict the attention to the parameterization specified in Sun (2008) and the conditions stated above. In the first period of each auditing cycle, the investors have perfect knowledge of the value of y^* given the revelation of previous earnings. In the second period, they form an expectation of actual earnings in period 1 based on the report in period 1 and the previously revealed earnings, and use this expectation to infer the current distribution of earnings for both compensation design purposes and firm valuation purposes.

The threshold level y^* can be derived as follows:

$$y^* = \begin{cases} \rho y + k & \text{in period 1,} \\ \rho \left[pr + (1-p)(r-a) \right] + k & \text{in period 2.} \end{cases}$$

[19] A possible alternative interpretation of the existence of threshold level outside the model is that executives strive to beat the consensus earnings forecast by financial analysts, and the best forecast is the conditional mean of earnings given the previous earnings reports.

Parameter	Value
ρ	0.77
k	0.23
a	2.1
β	0.98
F_1	31.8
F_2	$2F_1$

Table 4: Parameter values in the numerical example with continuous earnings

For the baseline case without earnings management ($x = 0$), reported earnings are always truthful, and the pricing function can be derived analytically. In this case, there is no difference between the reporting period (that is, period 1 of each auditing cycle) and the revelation period (that is, period 2 of each auditing cycle). The pricing equations in each period thus coincide with each other, equal to the sum of discounted expected future earnings.

$$\begin{aligned} q(y) &= E\Big\{(\rho y + k + \epsilon) + \beta\left[\rho(\rho y + k + \epsilon) + k + \epsilon\right] + \beta^2\{\rho\left[\rho(\rho y + k + \epsilon) + k + \epsilon\right] + k + \epsilon\} + \cdots\Big\} \\ &= \lim_{n\to\infty} \frac{\rho\left[1 - (\beta\rho)^n\right]}{} \end{aligned}$$

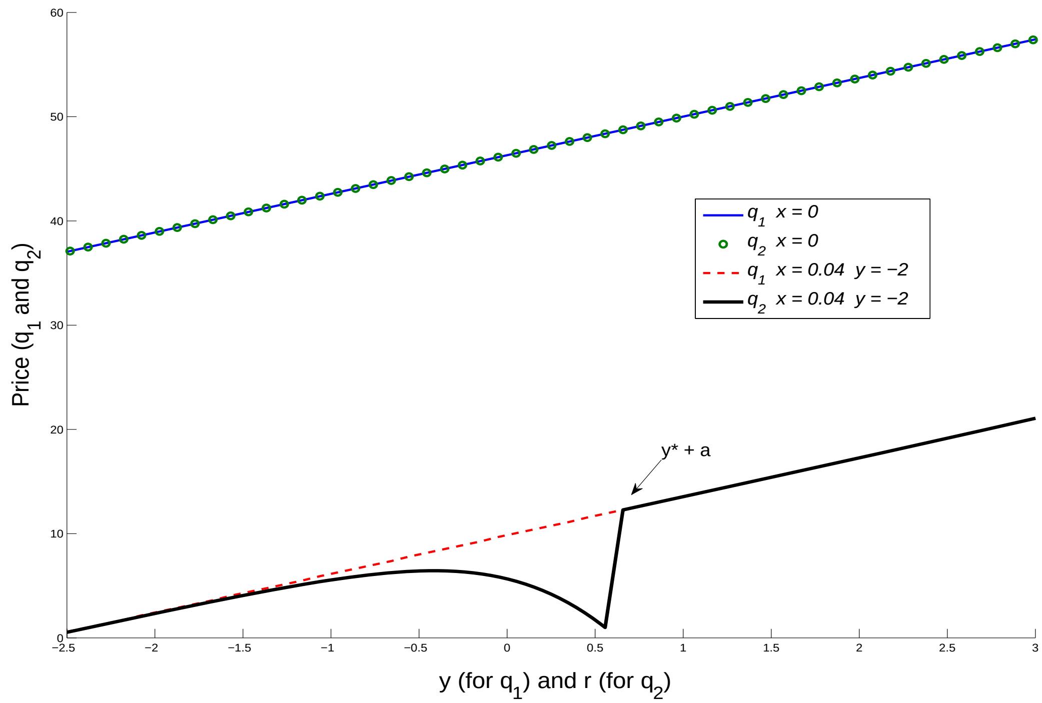

Figure 5: Pricing function with continuous earnings

value of x and F_1, compared with the value calibrated in the next section, to demonstrate the impact of earnings management on price dynamics.

Figure 5 shows how period 1's price varies with the revealed previous earnings as well as how period 2's price varies with the reported earnings, keeping the previously revealed earnings fixed. The dotted line and the light line that overlap with each other represent the price of period 1 (as a function of y) and that of period 2 (as a function of r) in the baseline case. The dashed line is period 1's price (as a function of y) with earnings management, and the dark line is period 2's price (as a function of r), for a given level of previous earnings y. Compared to the baseline case, a positive value of x makes the prices in both periods lower for a given level of previous earnings and earnings report. The price is discounted to reflect future monetary losses associated with restatement announcements because of the possibility of current period misreporting. The shift of prices is parallel (except for some deviation in period 2), because the possibility of having a false report in the current period is independent of y under the current assumptions.

With earnings management opportunity, the price of period 1 and that of period 2 differ only to reflect the additional information coming from the comparison between y and r. In period 2, the comparison between y and r reveals some information about the possibility that r is a false report, as shown in (26). Note that y^* is the conditional mean of the true earnings, which is a function of y. If r is very small, it is unlikely that the report has been inflated. If r is very large, it cannot be a manipulated report because there is no incentive for earnings management when the true earnings are greater than y^*. In particular, if $r > y^* + a$, the investors can infer (with probability 1) that r is a truthful report. In the medium range of r, the probability is large that r is a false report.

In the particular case with normal distributions of earnings, the following result can be shown.

L 4 *if* $r \in (-\infty, y^*]$ *or* $r \in (y^*, y^* + a)$, *p is strictly decreasing in y.*

P : See Appendix.

In Figure 6, period 2's price is plotted as a function of reported earnings for different levels of previously revealed earnings y. The dark, light, and dashed line represent a relatively low, medium, and high level of previous earnings respectively. If the previously revealed earnings are higher, the threshold level that induces truthful reporting is thus higher. The sharp drop-off of prices occurs at a higher level of reports.

5 Q

In this section I describe how I calibrate the model. Because this model describes individual stock returns, the calibration strategy is to simulate realizations of productivity shocks and earnings management opportunities for a large number of individual firms, gather the return sequences together, and then set the parameter values so as to match the aggregate targets.

To capture fluctuations in stock market indices, the calibrated model incorporates aggregate uncertainty: an aggregate productivity shock. The production process that individual firms follow is thus specified as $y' = \rho y + \epsilon_a + \epsilon_i$, where $\epsilon_a \sim N(0, \sigma_a^2)$ and $\epsilon_i \sim N(0, \sigma_i^2)$. Here, ϵ_a and ϵ_i represent aggregate productivity shock and idiosyncratic productivity shock

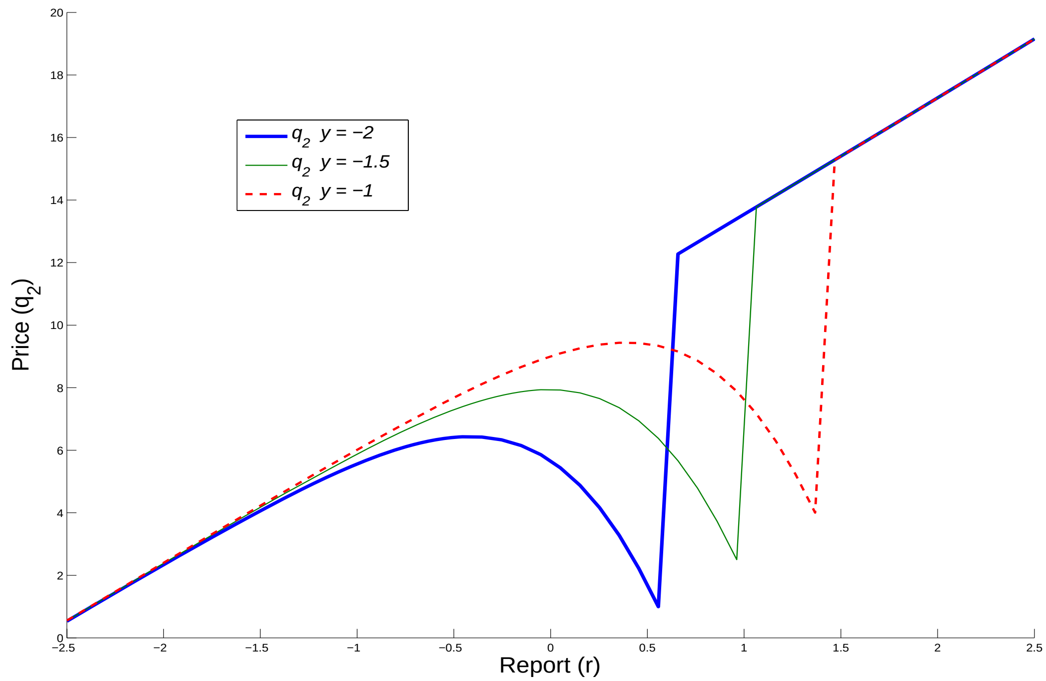

Figure 6: Pricing function in period 2

respectively, and they are independent. Aggregate productivity shock is assumed to be observable to both managers and investors. In doing so, I maintain the focus on the asymmetric information between managers and investors regarding idiosyncratic performance, without causing additional inference problems.[20]

Parameter	Description	Value
β	Discount factor	0.98
ρ	Autoregressive parameter	0.77
k	Constant drift	0.23
σ_a	Std.Dev of aggregate productivity shock	0.07
σ_i	Std.Dev of idiosyncratic productivity shock	0.11
x	Earnings management prevalence	0.04
a	Amount of overstatement	0.07
F_1	Monetary loss for one restatement	1.06
F_2	Monetary loss for two restatements	2.12

Table 5: Benchmark parameterization

bration. Second, counterfactual experiments are conducted by considering different levels of earnings management prevalence to assess the impact of earnings management in financial markets.

5.1 B[illegible]

Table 5 contains the benchmark parameter values. The period length is set to be half a year. The annual periodicity of restatements is thus in accordance with the empirical finding that the average number of restated fiscal quarters is about four (Wu, 2002).[22] The discount factor β is chosen to be 0.98 so that the implied semiannual real interest rate is 2 percent.

The autoregressive parameter

	Mean	Std.Dev	Autocorr	Std.Dev of avg. earnings
Scaled earnings	0.06	0.21	0.77	0.12

Table 6: Moments of semiannual scaled earnings

quarterly data item #69) to study firms' earnings. The results are also computed using earnings before extraordinary items (Compustat quarterly data item #8), and the results are generally consistent for these two alternative measures of earnings. The earnings data are drawn from a broad spectrum of firm sizes, and are therefore scaled following the approach in the literature. The earnings variable is scaled by beginning-of-the-period market value of common equity, computed as the close price in the end of the previous period multiplied by the number of common shares outstanding (i.e., [one-period-lagged Compustat quarterly data item #14] $\times$ [Compustat quarterly data item #61]). Following the convention, I also winsorize the data at 1 percent extreme values from each tail to reduce the impact of outliers and data errors.

The descriptive statistics of semiannual earnings in the sample are presented in Table 6. I normalize the steady-state level of actual earnings to be one, that is, $\bar{y} = \dfrac{k}{1-\rho} = 1$. The value of ρ is chosen to match with the average autocorrelation of firms' earnings, which is the third entry in Table 6. This gives $\rho = 0.77$, and $k = 1 - \rho = 0.23$. The standard deviation of aggregate productivity shock σ_a is set to be 0.07 to match with the time variation of average earnings across firms, shown in the fourth column in Table 6. As aggregate productivity shock and idiosyncratic productivity shock are independent of one another, given the variance of aggregate productivity shock, the standard deviation of idiosyncratic productivity shock is calculated to be $\sigma_i = 0.11$.

The parameter x is calibrated to be 0.04, yielding an overall earnings restatement rate 2 percent. This feature is in line with the average frequency of restatement announcements among publicly traded companies over the period of Jan 1997 to Sep 2005 (GAO, 2002 and GAO, 2006).[24] Wu (2002) documents that the average amount of restated earnings in her

[24]To identify and collect financial statements, GAO (2002, 2006) use Lexis-Nexis, an online periodical database, to conduct an intensive keyword search using variations of the word "restate." They include only announced restatements that were being made to correct previous material misstatements of financial results, while exclude announcements involving stock splits, changes in accounting principles, and other

sample is −\$9.8 million, while the average number of restated quarters is 4.2.[25] As the model is calibrated on a semiannual basis, I choose the amount of overstatement to be half of \$9.8 million in each period, that is, \$4.9 million. After scaled by average market value of listed companies and then normalized by average scaled earnings, a is 0.07.

To measure the monetary loss that the investors incur in the event of earnings restatements in the model, the current paper focuses on the average immediate market-adjusted loss in market capitalization of restating companies, that is, \$75.5 million for each restatement announcement (GAO, 2002 and GAO, 2006).[26] I choose the three-trading-day window to focus regarding the market response to the exclusion of other factors. This measure provides a lower bound for the financial losses the investors suffer from restatements, and the associated result serves as a lower bound for evaluating the importance of earnings management in financial markets. The scaled and normalized measure for the financial loss associated with each restatement is $F_1 = 1.06$. F_2 is then set to be 2.12.

5.2 Results

I report the simulation results on the parsimoniously parameterized model using the benchmark calibration for 500 firms and compare the statistical properties with S&P 500 index returns data. To get compound semiannual returns, I obtain S&P 500 quarterly returns from CRSP quarterly files from Jan 1931 to Dec 2007.[27]

Table 7 shows that relative to S&P 500 Index data, the volatility of the model-generated data is moderately lower. Table 8 compares EGARCH estimation results from the model returns and S&P 500 Index returns. The coefficients of the EGARCH (1,1) model are

financial statement restatements that were not made to correct mistakes in the application of accounting standards.

[25] Wu (2002) analyzes 932 earnings restatements from Jan 1997 through Dec 2001. The raw restated earnings magnitude runs from \$1.1 billion downward to \$470 million upward.

[26] To determine the immediate impact on stock prices, GAO (2002) analyzes 689 earnings restatements that were announced from January 1997 to March 2002. GAO (2006) examines 1061 restatement announcements from July 2002 to September 2005. For each of these cases, they examine the company's stock price on the trading days before, of, and after the announcement date to assess the immediate impact and calculate the change in market capitalization. I take an average of the immediate market-adjusted loss in market capitalization in the two samples.

[27] I consider a longer-period sample for stock returns than company earnings, excluding the 1929 stock market crash. The longer time span is chosen due to the semiannual frequency of the model.

	Standard Deviation
Model data	0.0714
S&P 500 data	0.1063

Table 7: Comparison of data volatility

Model data	Coefficient	Std.Error	t-statistic
K	-5.0000	2.5890	-1.9312
G	0.5260	0.2454	2.1436
A	0.0529	0.0235	2.2474
L	-0.0234	0.0139	-1.6784

S&P 500 data	Coefficient	Std.Error	t-statistic
K	-1.1029	0.4893	-2.2540
G	0.7365	0.1173	6.2789
A	0.3057	0.1542	1.9830
L	-0.2557	0.1127	-2.2703

Table 8: Comparison of EGARCH(1,1) estimation results
Variance equation: $\log \sigma_t^2 = K + G \log \sigma_{t-1}^2 + A[|\epsilon_{t-1}|/\sigma_{t-1} - E\{|\epsilon_{t-1}|/\sigma_{t-1}\}] + L[\epsilon_{t-1}/\sigma_{t-1}]$.

all statistically significant beyond the 95% confidence level. Consistent with the data, the conditional variance process is strongly persistent, although the magnitude of G coefficient is not as much as the data show. Since the coefficient L has a negative value, the model displays asymmetric volatility — negative surprises increase volatility more than positive surprises.

The intuition for the EGARCH effect in the binary example with two levels of earnings can be extended to the current model with a continuum of earnings. The general unifying story is that earnings management goes hand-in-hand with weak performance, because the financial incentive to artificially inflate earnings is strong when the earnings realization is poor. Relatively low earnings lead to more frequent future restatements than high earnings, generating greater movements in the return data. The return volatility becomes state-dependent, and the state (actual earnings) is persistent. Return volatility is thus persistent and asymmetric. In addition to this direct impact, an indirect effect due to *suspicion* of earnings management amplifies the persistence and asymmetry in return volatility. As shown in Figure 6, the possibility of earnings management creates a region of reports at the lower end that cause active learning and intensive suspicion of misstatement. Investors lower the price in anticipation of restatements. The uncertainty regarding the firm's fundamental value and subsequent outcomes is increased in this case, and some of the earnings reports under suspicion are associated with subsequent restatements and market fluctuations. Because the reported numbers tend to persist, the volatility also persists and exhibits asymmetry.

Although the model is consistent with volatility clustering and asymmetric volatility in the data, the magnitude is somewhat smaller. The A coefficient and L coefficient in S&P 500 Index returns are an order of magnitude greater than can be reproduced in the model. In light of the difficulties in measuring monetary losses in the event of earnings restatements, the discrepancy is not as large as it appears. For example, GAO (2002) and GAO (2006) show that restatement announcements have a negative effect on stock prices beyond their immediate impact. They find persistent market capitalization declines for restating companies. After controlling for the movement in the overall market, they report an average of \$79.3 million loss in market value from 20 trading days before through 20 trading days after a restatement announcement (the intermediate impact) and an average of \$136.1 million loss in market value from 60 trading days before through 60 trading days after the announcement (the longer-term impact). In addition, the use of market capitalization loss as a proxy for monetary loss that the investors incur precludes other potentially important factors.[28] The effects of such errors would be to bias the financial loss downwards, a correction of which would result in the model moving closer to the data. Measurement errors in the frequency of earnings management would have a similar effect on dynamic return patterns. Another plausible explanation for the discrepancy between model prediction and observational data is the oversimplicity of the model. Thus, although the overall fit of the model is good, it is not surprising, given the level of abstraction, that there are elements of the fine structure of returns the model is not designed to capture.

[28]For example, the loss of confidence in the corporate financial reporting could also hurt business and investment opportunities. Furthermore, the reduced availability and higher cost of capital may as well cause firms to postpone capital spending plans and accelerate layoffs. How to accurately measure the efficiency loss associated with earnings management is a question that warrants further research.

x=0	Coefficient	Std.Error	t-statistic
K	-4.9882	0.6962	-7.1651
G	0.0028	0.7555	0.0037
A	0.0116	0.0279	0.4133
L	0.0009	0.0007	1.2500

x=0.04	Coefficient	Std.Error	t-statistic
K	-5.0000	2.5890	-1.9312
G	0.5260	0.2454	2.1436
A	0.0529	0.0235	2.2474
L	-0.0234	0.0139	-1.6784

x=0.1	Coefficient	Std.Error	t-statistic
K	-2.9453	1.6996	-1.7330
G	0.6786	0.1855	3.6589
A	0.0353	0.0203	1.7393
L	-0.0255	0.0129	-1.9729

Table 9: EGARCH(1,1) estimation results with different levels of x
Variance equation: $\log \sigma_t^2 = K + G \log \sigma_{t-1}^2 + A[|\epsilon_{t-1}|/\sigma_{t-1} - E\{|\epsilon_{t-1}|/\sigma_{t-1}\}] + L[\epsilon_{t-1}/\sigma_{t-1}]$.

5.3 [illegible]

GAO (2002) and GAO (2006) document a significant upward trend in the number of restatements over time. To gain insight on policy-related issues, it is of interest to examine how the magnitude of financial anomalies varies with the extent of earnings management. Here, I consider the economies with different levels of earnings management prevalence. Specifically, I consider various values of x to assess the importance of earnings management. In these economies with different values of x, the other parameters are chosen to match the same aggregate targets as in the benchmark calibration.

Table 9 presents the results. The extreme case of $x = 0$ in this model, shown in the first panel, corresponds to the standard Lucas asset-pricing model. In this case, earnings management does not exist. The estimated EGARCH coefficients are substantially reduced and insignificant. No long-memory persistence or asymmetric behavior is present in the model data.

As x is increased to 0.04 as in the calibrated model, the EGARCH estimation results on

x	Standard Deviation
0	0.0424
0.04	0.0714
0.1	0.1044

Table 10: Volatility of the model returns with different level of x

the simulated return data demonstrate the presence of strong persistence and asymmetry in volatility. When $x = 0.1$, G and L coefficients become larger in magnitude and more significant. These are strong indications that incorporating earnings management intensifies both persistence and asymmetry in return volatility.

Table 10 contains the standard deviation of returns in the simulated data. Consistent with the empirical studies mentioned in Section 1 and Section 3.2, as x increases (implying that the informativeness of earnings reports becomes weakened), the returns exhibit greater volatility. Monetary penalties charged upon restatement announcements generate large swings in the return sequence, and hence raise volatility.

Models such as the one considered in this paper can be used to predict the consequence of a particular corporate governance rule on financial reporting. The comparison of the financial returns dynamics with different prevalence of earnings management underscores why earnings management is of central importance in pricing of financial assets, in understanding the risk implied by empirical anomalies, and in the current debate about advantages of strict implementation of corporate governance policy, such as the Sarbanes-Oxley Act.

6 Robustness Check

In this section, robustness check of the baseline model is conducted, both in terms of quantitative evaluations and model specifications. First, following an alternative calibration strategy, I recalibrate the model to Compustat Unrestated data, and study the return patterns. Second, I consider a setting in which investigations are conducted stochastically, and check whether model dynamics are robust to a stochastic feature of revelations.

	Mean	Std.Dev	Autocorr	Std.Dev of avg.	Avg. of Std.Dev
Scaled reports	0.10	0.22	0.82	0.03	0.15

Table 11: Moments of semiannual scaled reports

Parameter	Description	Value
ρ	Autoregressive parameter	0.82
k	Constant drift	0.18
σ_a	Std.Dev of aggregate productivity shock	0.02
σ_i	Std.Dev of idiosyncratic productivity shock	0.08
β	Discount factor	0.98
x	Earnings management prevalence	0.04
a	Amount of overstatement	0.03
F_1	Monetary loss for one restatement	0.49
F_2	Monetary loss for two restatements	0.98

Model data	Coefficient	Std.Error	t-statistic
K	-2.7503	1.4893	-1.8467
G	0.8049	0.1056	7.6225
A	0.0339	0.0166	2.0492
L	-0.0231	0.0106	-2.1911

S&P 500 data	Coefficient	Std.Error	t-statistic
K	-1.1029	0.4893	-2.2540
G	0.7365	0.1173	6.2789
A	0.3057	0.1542	1.9830
L	-0.2557	0.1127	-2.2703

Table 13: Comparison of EGARCH(1,1) estimation results
Variance equation: $\log \sigma_t^2 = K + G \log \sigma_{t-1}^2 + A[|\epsilon_{t-1}|/\sigma_{t-1} - E\{|\epsilon_{t-1}|/\sigma_{t-1}\}] + L[\epsilon_{t-1}/\sigma_{t-1}]$.

	Standard Deviation
Model data	0.0300
S&P 500 data	0.1063

Table 14: Comparison of data volatility

calibration, and that gives $\beta = 0.98, x = 0.04, a = 0.03, F_1 = 0.49$, and $F_2 = 0.98$, as presented in Table 12. Some values are different from the benchmark calibration because of the normalization of reported earnings to unity, compared with the normalization of restated earnings to unity.

Table 13 contains measures of EGARCH effect for the model returns and S&P 500 Index returns. The results are similar to those with the benchmark parameterization, except that the G coefficient somewhat overshoots. The stronger persistence in volatility than in the benchmark calibration is attributable to the higher persistence in firms' earnings. This result confirms that most of the volatility clustering in the model has to come from the persistent component in earnings management, which directly stems from the persistent component in earnings. This element of the model is crucial in making it consistent with the observed heteroskedasticity. The finding that EGARCH effect is quite similar for different calibration strategies suggests that, even though the parameters may differ across economies, the nature of return dynamics can still be quite similar.

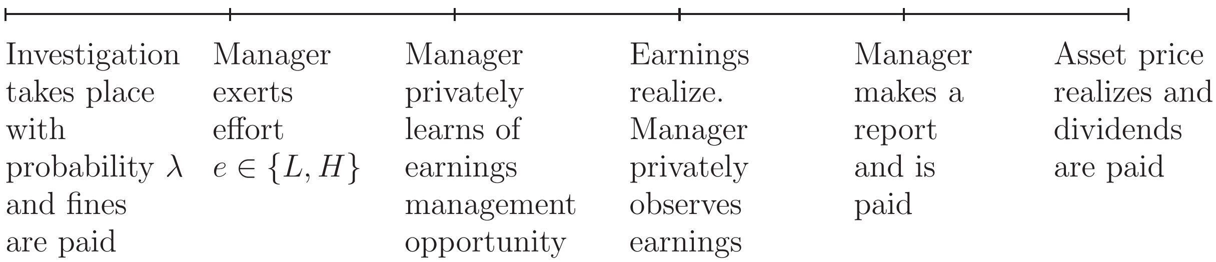

Figure 7: Model timeline with stochastic investigation

Table 14 compares the volatility of the model and the data. Compared with the benchmark parameterization, the model volatility is reduced. The reason is that the value of monetary loss associated with earnings management is calibrated to be lower (in particular, less than half in size), leading to a more moderate reaction of asset returns to restatement announcements. A smaller fluctuation of the returns during restatements produces lower volatility.

6.2 [illegible]

In the baseline model, the periodic investigation is conducted deterministically every two periods. To examine how this assumption affects the results, here I consider a setting where investigations take place stochastically. As in Section 2 and Section 3, there are two levels of earnings: $y \in \{l, h\}$. Actual earnings follow a Markov process

$$\Pr(y_{t+1} = j | y_t = i) = \pi_{ij}, \qquad \forall i \in \{l, h\}, \quad \forall j \in \{l, h\}.$$

The investigation regarding financial reporting is now assumed to be stochastic, and occurs with probability λ every period. If the investigation takes place, all the previous earnings since the most recent investigation are revealed. The financial statements in the corresponding periods when earnings management occurs have to be restated, and the investors bear monetary penalties. More specifically, the amount of financial charges upon restatement announcements is a strictly increasing function of the number of periods in which the manager manipulates earnings. The timeline of the model events in each period is described in Figure 7.

Note that the derivation of the posterior probability of having a false report at each point in time requires utilizing the entire history of reports since the most recent investigation up to the current report. In particular, when the manager makes an earnings announcement every period, the investors not only infer the current realization and predict future earnings, but also revise their expectation on each previous report in history.

Fortunately, in this setting all the relevant information in the reporting history can be summarized with a small set of state variables. In what follows, the problem is reduced to a variational problem in which history dependence can be summarized and asset price can be characterized by the following five state variables.[29]

- γ: the conditional probability (with the information from the current report) that the current true earnings are high;
- Z: the expected number of periods involving earnings management since the last investigation until the most recent low report ($Z = 0$ if there is no low report since the last investigation until the previous period);
- N: the number of consecutive high reports until the previous period since the last low report or the last investigation, whichever is more recent;
- r: the current earnings report, $r \in \{\tilde{l}, \tilde{h}\}$;
- $\bar{y}$: the true earnings before the series of consecutive N high reports starts.

Given the earnings management incentive in this binary setting, the current true earnings are revealed under two circumstances. The first is when the investigation regarding financial reporting takes place. In this case, the entire history of earnings realizations is revealed. The second is when the manager sends a low report. If the reported earnings are low, although the credibility of financial statements in prior periods remains ambiguous, the current earnings are low with certainty. In the following, I derive the pricing functions that describe a stationary solution to the problem using these state variables. The stock price at time t is denoted by $q_t = P(\gamma_t, Z_t, N_t, r_t, \bar{y}_t)$.

Let the monetary penalties charged for earnings management be a linear function of the number of restating periods upon investigation. Specifically, the fines $F = \kappa n$, where κ is

[29] For detailed examples of what each state variable represents, see Appendix B.

a constant and n is the number of periods involving earnings management since the most recent investigation. As the investors update their beliefs in the standard Bayesian fashion, γ' evolves following Bayes' Rule:

$$\gamma' = \begin{cases} \dfrac{\gamma\pi_{hh} + (1-\gamma)\pi_{lh}}{\gamma\pi_{hh} + (1-\gamma)\pi_{lh} + \gamma(1-\pi_{hh})x + (1-\gamma)(1-\pi_{lh})x}, & r = \tilde{h} \text{ at } t+1, \\ 0, & r = \tilde{l} \text{ at } t+1. \end{cases}$$

First, the price associated with a high report, $P(\gamma, Z, N, \tilde{h}, \bar{y})$, is derived.[30]

$$P(\gamma, Z, N, \tilde{h}, \bar{y}) = \tilde{h} + \beta\left[(1-\lambda)W_n^{\tilde{h}} + \lambda W_i^{\tilde{h}}\right]. \tag{28}$$

Here, β is the discount factor. $W_n^{\tilde{h}}$ represents the expected price if the investigation does not occur in the beginning of the next period, and $W_i^{\tilde{h}}$ represents the expected price if the investigation occurs. Both prices are conditional on a current high report.

If the investigation does not take place in the beginning of the next period, the expected price is

$$W_n^{\tilde{h}} = \mu P(\gamma', Z, N+1, \tilde{h}, \bar{y}) + (1-\mu)P(0, Z, N+1, \tilde{l}, \bar{y}). \tag{29}$$

The first term in (29) is the expected price if the next report is high. The second term is the expected price when the report in the next period is low. Note that a low report is always truthful, and thus γ is updated to 0. μ denotes the conditional probability that the manager makes a high report in the next period:

$$\mu = \gamma\pi_{hh} + \gamma(1-\pi_{hh})x + (1-\gamma)\pi_{lh} + (1-\gamma)(1-\pi_{lh})x$$

If the investigation takes place in the next period, the expected price is

$$\begin{aligned} W_i^{\tilde{h}} = &-\kappa[Z + f(N+1; \bar{y}))] \\ &+ \gamma\left[\xi_1 P\left(\frac{\pi_{hh}}{\xi_1}, 0, 0, \tilde{h}, h\right) + (1-\xi_1)P(0,0,0,\tilde{l}, h)\right] \\ &+ (1-\gamma)\left[\xi_2 P\left(\frac{\pi_{lh}}{\xi_2}, 0, 0, \tilde{h}, l\right) + (1-\xi_2)P(0,0,0,\tilde{l}, l)\right]. \end{aligned} \tag{30}$$

[30] Again, the impact of wage values in price calculations is not considered in the current analysis.

where ξ_1 represents the conditional probability of having a high report in the next period, given the current true earnings are high. ξ_2 is the probability of having a high report conditional on that the current true earnings are low.

$$\xi_1 = \pi_{hh} + (1 - \pi_{hh})x$$

$$\xi_2 = \pi_{lh} + (1 - \pi_{lh})x$$

The first term in (30) is the expected amount of financial penalties for earnings management. $f(N+1;\bar{y})$ denotes the expected number of falsified reports among the $(N+1)$ consecutive reports of high earnings since the last low report or the last investigation, whichever is more recent. The function $f(N+1;\bar{y})$ is calculated from the model fundamental in a recursive manner, and the method is illustrated in Appendix C. The number of the expected restating periods is thus the sum of $f(N+1;\bar{y})$ and the expected number of periods involving earnings management from the last investigation through the most recent low report, Z. Recall that γ is the conditional probability that the current high report is truthful. The second term in (30) thus represents the expected price if the current high report is truthful. The third term is the case in which the current earnings are low and have been overstated.

Now let us consider the asset price if the current report is low.

$$P(0, Z, N, \tilde{l}, \bar{y}) = \tilde{l} + \beta \left[(1 - \lambda) W_n^{\tilde{l}} + \lambda W_i^{\tilde{l}} \right]. \tag{31}$$

where $W_n^{\tilde{l}}$ and $W_i^{\tilde{l}}$ represent the expected price if the investigation does not occur in the next period and the expected price if the investigation occurs, respectively, conditional on a current low report.

If the investigation does not take place in the next period, the expected price is

$$W_n^{\tilde{l}} = \xi P\left(\frac{\pi_{lh}}{\xi}, Z, 0, \tilde{h}, l\right) + (1 - \xi) P(0, Z, 0, \tilde{l}, l)$$

where ξ denotes the conditional probability that the manager makes a high report in the next period:

$$\xi = \pi_{lh} + (1 - \pi_{lh})x$$

Parameter	Value
h	20
l	10
π_{hh}	0.8
π_{ll}	0.8
β	0.98
κ	15
λ	0.5

Table 15: Parameter values in the numerical example with binary earnings

If the investigation takes place in the next period:

$$\begin{aligned} W_i^{\tilde{l}} = & -\kappa[Z + f(N;\bar{y})] \\ & + \xi P\left(\frac{\pi_{lh}}{\xi}, 0, 0, \tilde{h}, l\right) \\ & + (1-\xi)P\left(0, 0, 0, \tilde{l}, l\right) \end{aligned} \tag{32}$$

The first term in (32) is the expected monetary charges for earnings management, which is a linear function of the expected number of restating periods. The second term is the expected price if the realization of actual earnings is high in the next period, and the third term corresponds to the case in which the realization is low. Thus, from (28) and (31), the price in each period can be solved recursively.

Table 15 contains the parameter values. The pricing functions are computed numerically. Figure 8 displays $f(N, \bar{y})$, the shape of which may vary with parameterizations. Figure 9 and Figure 10 show how the prices associated with a high report change with γ and N. As the monetary penalties associated with earnings management is a linear function of the number of restated financial statements, the price in response to a high report is linearly increasing in γ and linearly decreasing in Z. As shown in Figure 11, the price in response to a low report is also linearly decreasing in both Z, with γ updated to 0.

The model is simulated for 10,000 periods. in a numerical example. In order to illustrate the influence of earnings management incentive on dynamic return patterns, I compare the model returns with $x = 0$ and those with $x = 0.1$. Table 16 presents the EGARCH estimation results on the model returns. In a model without earnings management ($x = 0$), there is

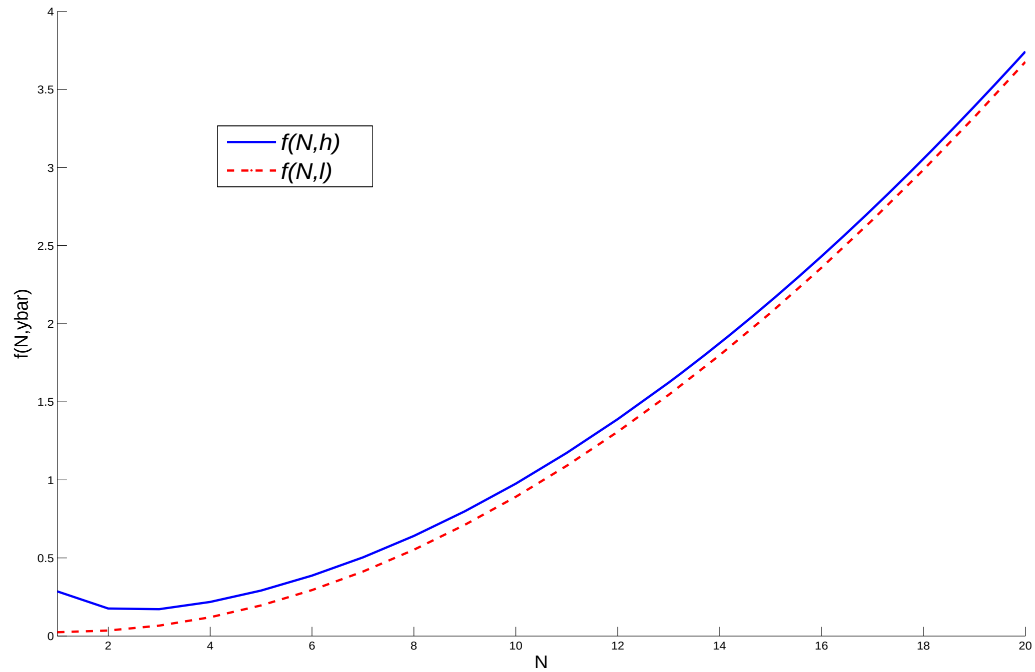

Figure 8: The expected number of inflated reports among N consecutive high reports $f(N, \bar{y})$

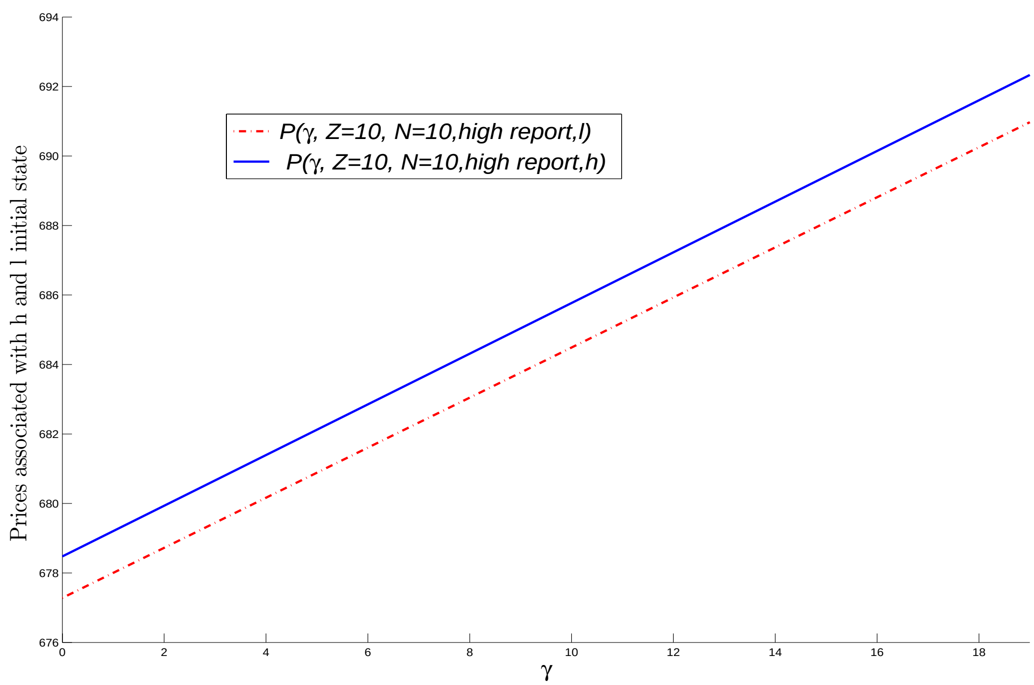

Figure 9: Price for a high report as a function of γ

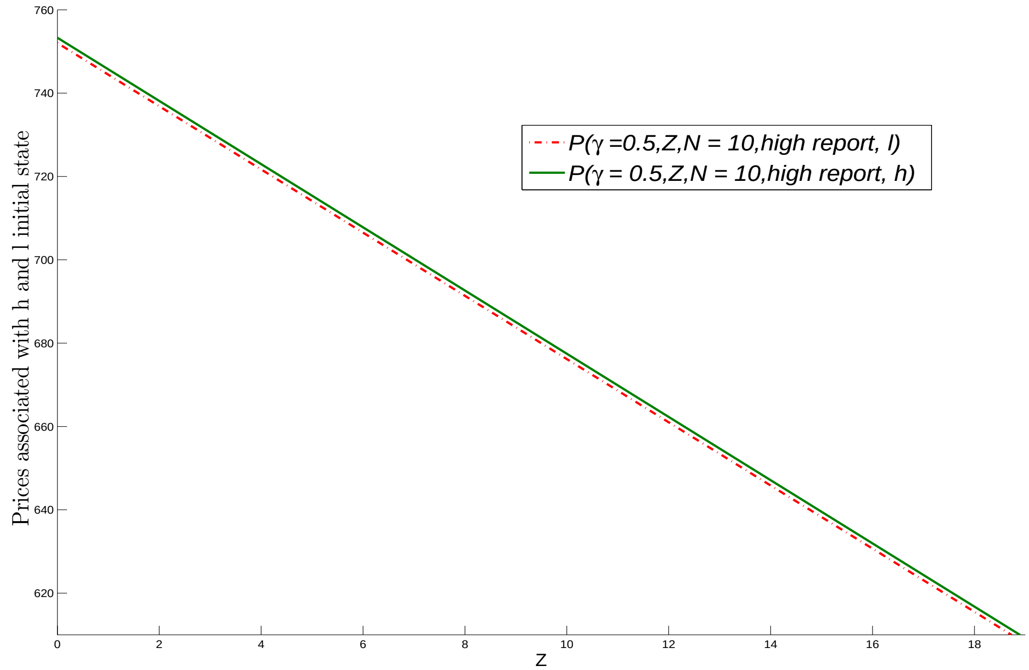

Figure 10: Price for a high report as a function of Z

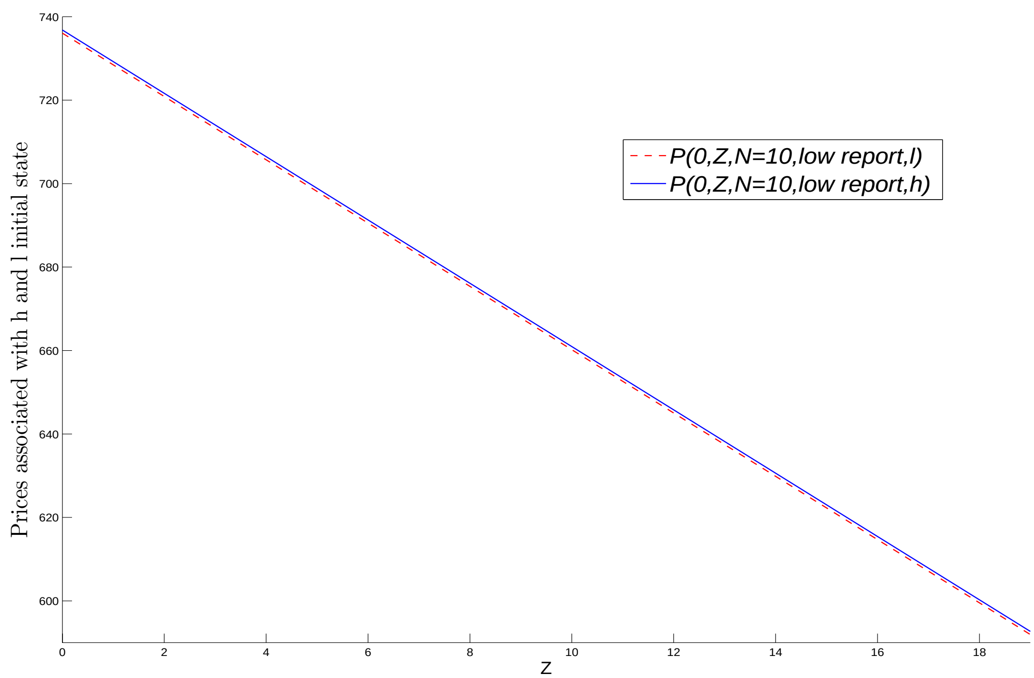

Figure 11: Price for a low report as a function of Z

x=0	Coefficient	Std.Error	t-statistic
K	-5.0000	12.8300	-0.3897
G	0.0576	0.0880	0.6552
A	0.0033	0.0119	0.2838
L	0.0041	0.0066	0.6195

x=0.1	Coefficient	Std.Error	t-statistic
K	-2.0291	0.2979	-6.8092
G	0.7441	0.0376	19.7951
A	0.1068	0.0207	5.1616
L	-0.0841	0.0197	-4.2789

Table 16: EGARCH(1,1) estimation results
Variance equation: $\log \sigma_t^2 = K + G \log \sigma_{t-1}^2 + A[|\epsilon_{t-1}|/\sigma_{t-1} - E\{|\epsilon_{t-1}|/\sigma_{t-1}\}] + L[\epsilon_{t-1}/\sigma_{t-1}]$

x	Standard Deviation
0	0.0134
0.1	0.0193
0.2	0.0201

Table 17: Volatility of the model returns

no persistence in return volatility (shown in the upper panel). As earnings management becomes possible, the coefficients of the EGARCH model are all statistically significant. Persistence and asymmetry are present in the model return volatility. In addition, Table 17 shows that the model returns become more volatile as x increases. The same set of results and intuition from the model with deterministic monitoring carry through.

This model of stochastic investigation assumes a constant exogenous probability of monitoring in every period. However, with a positive monitoring cost, it is natural to argue that monitoring would occur with a higher probability in bad times, since there tends to be little interest in investigating when the market is booming. Accounting fraud does come in waves, and is detected more intensively during market collapses. As monitoring occurs more often when the aggregate state of the economy is bad and earnings management is more prevalent, the asymmetric behavior in stock returns tends to be more pronounced. An monitoring probability that varies with the aggregate economic prospects would strengthen the mechanism illustrated in this paper, and intensify these observed features of asset returns.

7 C s

This paper examines dynamic asset return patterns in an economy in which information about underlying profitabilities is obscured. An important ingredient in the current formulation of the asset-pricing problem is that executives intentionally manipulate financial information in their own best interests. Executives possess two dimensions of private information: realizations of actual earnings and realizations of earnings management opportunity. Because different combinations of these two could generate identical earnings reports, there is no strict monotonicity and hence no invertibility of the reporting function. Although the investors are fully rational, and they learn in a standard Bayesian fashion, they cannot perfectly filter out the manipulation component in the reports. Therefore, earnings management causes a pricing distortion — honest firms are undervalued, while firms that manipulate their accounting numbers are overpriced.

this paper also presents a likely source of non-fundamental volatility and financial risk.

The model of shareholders-manager behavior in an asset-pricing model presented here has been simplified in many ways. In particular, the current analysis does not explicitly model how the manager finances the discrepancy in the reports. As elaborated earlier in this paper,[31] leaving the source of funds outside the model is for the purpose of simplification without causing modeling inconsistency. Furthermore, this formulation can also be viewed as a simple way of illustrating the idea that the manager can divert resources from profitable investment to current payouts. Fully formulating this idea requires a production economy with investment, and I take the current framework as the first step towards the ultimate goal. Future research will develop a production-based model to scrutinize the capital allocation problem caused by earnings management, and evaluate its economic costs.

[31]See footnote 8 in Section 2.

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After the parallel shift of $f(y|e = H)$ and $f(y|e = L)$ by δ, the conditional distribution of actual earnings given effort is denoted by $g(y|e = i) = f(y - \delta|e = i)$, $\forall i \in \{L, H\}$. The principal has a utility function given by $V(y - w)$.

The Lagrangian for the principal's problem in this case is

$$\mathcal{L} = \int_{\underline{y}+\delta}^{\bar{y}+\delta} \Big\{ V\left[y - w(r(y))\right] g(y|e = H) + \lambda \left[u(w)g(y|e = H) - \bar{U}\right] \\ + \mu \left[u(w)g(y|e = H) - u(w)g(y|e = L) - c\right] \Big\} dy$$

The reporting function $r(y)$ is given by

$$r(y) = \begin{cases} y + a & \text{if } u\left[w(y + a)\right] - u\left[w(y)\right] > \psi, \text{ and earnings management opportunity realizes} \\ y & \text{otherwise.} \end{cases}$$

Differentiating with respect to $w(r)$ inside the integral sign, we obtain the first-order condition. Assuming that it is optimal to elicit high effort, an optimal incentive compensation scheme $w(r)$ satisfies

$$\frac{V'[y - w(r)]}{u'[w(r)]} = \lambda + \mu \left[1 - \frac{g\left(y(r)|e = L\right)}{g\left(y(r)|e = H\right)}\right], \tag{33}$$

Assume that the principal is risk-neutral, and the manager's utility function takes the logarithm form given by $u(w) = \log(w)$. (33) simplifies to

$$\begin{aligned} w(r) &= \lambda + \mu \left[1 - \frac{g\left(y(r)|e = L\right)}{g\left(y(r)|e = H\right)}\right] \\ &= \lambda + \mu \left[1 - \frac{f\left(y(r) - \delta|e = L\right)}{f\left(y(r) - \delta|e = H\right)}\right] \end{aligned} \tag{34}$$

The solutions also satisfy the complementary slackness conditions

$$\lambda \int_{\underline{y}+\delta}^{\bar{y}+\delta} \left\{u(w)g(y|e = H) - \bar{U}\right\} dy = 0,$$

$$\mu \int_{\underline{y}+\delta}^{\bar{y}+\delta} \{u(w)g(y|e=H) - u(w)g(y|e=L) - c\}\, dy = 0.$$

which can be rewritten as

$$\lambda \int_{\underline{y}+\delta}^{\bar{y}+\delta} \left\{u(w)f(y-\delta|e=H) - \bar{U}\right\} dy = 0, \tag{35}$$

$$\mu \int_{\underline{y}+\delta}^{\bar{y}+\delta} \{u(w)f(y-\delta|e=H) - u(w)f(y-\delta|e=L) - c\}\, dy = 0. \tag{36}$$

The following inequalities should also be satisfied

$$\lambda \geq 0, \quad \mu \geq 0. \tag{37}$$

Let $w^*(r)$ be the solution to the principal's problem before the parallel shift of $f(y|e=H)$ and $f(y|e=L)$. λ^* and μ^* are the corresponding Lagrangian multipliers. Then $w^*(r)$, λ^*, and μ^* satisfy the first-order condition

$$w^*(r) = \lambda^* + \mu^* \left[1 - \frac{f\left(y(r)|e=L\right)}{f\left(y(r)|e=H\right)}\right]$$

together with the complementary slackness conditions

$$\lambda^* \int_{\underline{y}}^{\bar{y}} \left\{u(w)f(y|e=H) - \bar{U}\right\} dy = 0,$$

$$\mu^* \int_{\underline{y}}^{\bar{y}} \{u(w)f(y|e=H) - u(w)f(y|e=L) - c\}\, dy = 0.$$

and the inequalities

$$\lambda^* \geq 0, \quad \mu^* \geq 0.$$

It follows that

$$w^*(r-\delta) = \lambda^* + \mu^* \left[1 - \frac{f\left(y(r)-\delta|e=L\right)}{f\left(y(r)-\delta|e=H\right)}\right]$$

$$\lambda^* \int_{\underline{y}+\delta}^{\bar{y}+\delta} \left\{u\left[w^*(r-\delta)\right] f(y-\delta|e=H) - \bar{U}\right\} dy = 0,$$

$$\mu^* \int_{\underline{y}+\delta}^{\bar{y}+\delta} \left\{u\left[w^*(r-\delta)\right] f(y-\delta|e=H) - u\left[w^*(r-\delta)\right] f(y-\delta|e=L) - c\right\} dy = 0.$$

$$\lambda^* \geq 0, \quad \mu^* \geq 0.$$

It is straightforward to determine that $w(r) = w^*(r - \delta)$, $\lambda = \lambda^*$, and $\mu = \mu^*$ satisfy (34), (35), (36), and (37). The reporting choice $r(y)$ remains unchanged in this case. Therefore, a parallel shift of the wage function by δ solves the principal's problem. □

The idea underlying this analysis is that given a parallel shift of conditional distributions of output, a parallel shift of the wage payment schedule by the same amount provides the same incentive to the manager and same marginal value of effort to the risk-neutral principal. First, because the distribution of wage payment remains unchanged after parallel shifts of the wage function and output distribution by a same amount, the manager does not have an incentive to deviate from the recommended effort and reporting choice.

Second, the risk-neutral principal designs the compensation based on the monetary value of high effort relative to low effort, which is the difference in the residuals. The residual is the expected earnings net of compensation payment, conditional on high and low effort. The monetary value of effort can be denoted by $\Big[$(expected earnings given high effort - expected payment given high effort) $-$ (expected earnings given high effort $-$ expected payment given low effort)$\Big]$. It can be rewritten as $\Big[$(expected earnings given high effort $-$ expected earnings given low effort) $-$ (expected payment given high effort $-$ expected payment given low effort)$\Big]$. As long as $(\mu_H - \mu_L)$ and the wage distribution remain constant, the principal does not have any incentives to change the shape of incentive schemes.

A parallel shift of the wage schedule by an equal amount as the shift of output distributions provides the manager with the same incentive and the principal with the same value, and therefore is an optimal contract in this case.

Proof of Lemma 4:

If $r \in (y^*, y^* + a)$,

$$\begin{aligned}
p &= \Pr[y' = r | y] \\
&= \frac{f(r-k-\rho y)}{f(r-k-\rho y) + x f(r-a-k-\rho y)} \\
&= \frac{1}{1 + x \left[\frac{f(r-a-k-\rho y)}{f(r-k-\rho y)} \right]} \\
&= \frac{1}{1 + x \exp\left[\frac{1}{2\sigma}(r-k-\rho y)^2 - \frac{1}{2\sigma}(r-a-k-\rho y)^2 \right]} \\
&= \frac{1}{1 + x \exp\left[\frac{a}{2\sigma}(2r - 2k - 2\rho y - a) \right]}
\end{aligned}$$

Using the same property of normal distributions, it is straightforward to check that p is decreasing in r when $r < y^*$.

$$p = \frac{1}{1 + (1-x) \exp\left[\frac{a}{2\sigma}(2r - 2k - 2\rho y - a) \right]}.$$

□

To be clear on what each variable represents, a set of clarifying examples is provided in the following. Now let today be $t = 10$ and let the last investigation happen at the beginning of $t = 5$. Suppose that the true state of $t = 4$ is revealed to be y_4.

- If $\{r_5, r_6, r_7, r_8, r_9, r_{10}\} = \{\tilde{h}, \tilde{h}, \tilde{h}, \tilde{l}, \tilde{h}, \tilde{h}\}$, then, at $t = 10$, Z is the expected number of inflated reports during periods 5, 6, and 7; $N = 1$ (it does not include the current period); and $r = \tilde{h}$. $\bar{y} = l$, because the true state in period 8 is known to be low (recall that all the low reports are honest reports).

- If $\{r_5, r_6, r_7, r_8, r_9, r_{10}\} = \{\tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}\}$, then, at $t = 10$, $Z = 0$ (there is not any low report after the last investigation until the previous period); $N = 5$ (it does not include the current period); and $r = \tilde{h}$. $\bar{y} = y_4$, because its

- If r

- If $r_5 = \tilde{l}$, then $Z = 0$, $N = 0$, $r = \tilde{l}$, and $\bar{y} = y_4$.

C C $f(N;\bar{y})$

Let the information set $\mathcal{R}_N^{\bar{y}} \equiv \{\bar{y}, r_1 = \tilde{h}, r_2 = \tilde{h}, \cdots, r_N = \tilde{h}\}$. y_n represents the true earnings in period n, $\forall n \in \{1, 2, \cdots, N\}$. Thus $f(N;\bar{y})$ can be written as

$$\begin{aligned} f(N;\bar{y}) = & \Pr[y_1 = l|\mathcal{R}_N^{\bar{y}}] + \Pr[y_2 = l|\mathcal{R}_N^{\bar{y}}] + \cdots \\ & + \Pr[y_n = l|\mathcal{R}_N^{\bar{y}}] + \cdots + \Pr[y_N = l|\mathcal{R}_N^{\bar{y}}] \end{aligned}$$

The problem of deriving $f(N;\bar{y})$ in a recursive way is transformed into an equivalent problem, that is, to recursively derive

$$\Pr[y_n = l|\mathcal{R}_N^{\bar{y}}] = 1 - \Pr[y_n = h|\mathcal{R}_N^{\bar{y}}], \qquad \forall n \in \{1, 2, \cdots, N\}.$$

Note that

$$\begin{aligned} \mathcal{R}_N^h &\equiv \{h, r_1 = \tilde{h}, r_2 = \tilde{h}, \cdots, r_N = \tilde{h}\} \\ \mathcal{R}_N^l &\equiv \{l, r_1 = \tilde{h}, r_2 = \tilde{h}, \cdots, r_N = \tilde{h}\} \end{aligned}$$

The proof includes two steps. In step 1, $\Pr[y_1 = h|\mathcal{R}_1^l]$ and $\Pr[y_1 = h|\mathcal{R}_1^h]$ are calculated. In step 2, I show that $\Pr[y_n = h|\mathcal{R}_{N+1}^l]$ and $\Pr[y_n = h|\mathcal{R}_{N+1}^h]$, $\forall n \in \{1, 2, \cdots, N+1\}$, can be calculated using $\Pr[y_n = h|\mathcal{R}_N^l]$ and $\Pr[y_n = h|\mathcal{R}_N^h]$, $\forall n \in \{1, 2, \cdots, N\}$.

As the first step, $\Pr[y_1 = h|\mathcal{R}_1^l]$ and $\Pr[y_1 = h|\mathcal{R}_1^h]$ are derived as follows.

$$\begin{aligned} \Pr[y_1 = h|\mathcal{R}_1^l] &= \Pr[y_1 = h|\bar{y} = l, r_1 = \tilde{h}] \\ &= \frac{\Pr[y_1 = h, r_1 = \tilde{h}|\bar{y} = l]}{\Pr[r} \end{aligned}$$

In step 2, I first show that $\Pr[y_n = h|\mathcal{R}^l_{N+1}]$ can be calculated if $\Pr[y_n = h|\mathcal{R}^l_N]$ is known. For $n \in \{1, 2, \cdots, N+1\}$,

$$\Pr[y_n = h|\mathcal{R}^l_N, r_{N+1} = \tilde{h}] = \frac{\Pr[y_n = h, r_{N+1} = \tilde{h}|\mathcal{R}^l_N]}{\Pr[r_{N+1} = \tilde{h}|\mathcal{R}^l_N]}. \tag{38}$$

The denominator in (38), $\Pr[r_{N+1} = \tilde{h}|\mathcal{R}^l_N]$, is derived as the following.

$$\begin{aligned}\Pr[r_{N+1} = \tilde{h}|\mathcal{R}^l_N] =& \Pr[r_{N+1} = \tilde{h}, y_{N+1} = h|\mathcal{R}^l_N] + \Pr[r_{N+1} = \tilde{h}, y_{N+1} = l|\mathcal{R}^l_N] \\ =& \Pr[r_{N+1} = \tilde{h}|y_{N+1} = h, \mathcal{R}^l_N] \times \Pr[y_{N+1} = h|\mathcal{R}^l_N] \\ &+ \Pr[r_{N+1} = \tilde{h}|y_{N+1} = l, \mathcal{R}^l_N] \times \Pr[y_{N+1} = l|\mathcal{R}^l_N] \\ =& \Pr[y_{N+1} = h|\mathcal{R}^l_N] + x\left[1 - \Pr[y_{N+1} = h|\mathcal{R}^l_N]\right],\end{aligned}$$

where

$$\begin{aligned}\Pr[y_{N+1} = h|\mathcal{R}^l_N] =& \Pr[y_{N+1} = h, y_N = h|\mathcal{R}^l_N] + \Pr[y_{N+1} = h, y_N = l|\mathcal{R}^l_N] \\ =& \Pr[y_{N+1} = h|y_N = h, \mathcal{R}^l_N] \times \Pr[y_N = h|\mathcal{R}^l_N] \\ &+ \Pr[y_{N+1} = h|y_N = l, \mathcal{R}^l_N] \times \Pr[y_N = l|\mathcal{R}^l_N] \\ =& \pi_{hh} \Pr[y_N = h|\mathcal{R}^l_N] + \pi_{lh}\left[1 - \Pr[y_N = h|\mathcal{R}^l_N]\right].\end{aligned} \tag{39}$$

As $\Pr[y_N = h|\mathcal{R}^l_N]$ is known from the supposition, this can be calculated. The denominator is obtained

$$\begin{aligned}\Pr[r_{N+1} = \tilde{h}|\mathcal{R}^l_N] =& \pi_{hh} \Pr[y_N = h|\mathcal{R}^l_N] + \pi_{lh}\left[1 - \Pr[y_N = h|\mathcal{R}^l_N]\right] \\ &+ x\{1 - \pi_{hh} \Pr[y_N = h|\mathcal{R}^l_N] - \pi_{lh}\left[1 - \Pr[y_N = h|\mathcal{R}^l_N]\right]\}.\end{aligned} \tag{40}$$

Now let us consider the numerator in (38). For $n = N+1$, $\Pr[y_{N+1} = h, r_{N+1} = \tilde{h}|\mathcal{R}^l_N]$ can be rewritten as

$$\begin{aligned}\Pr[y_{N+1} = h, r_{N+1} = \tilde{h}|\mathcal{R}^l_N] &= \Pr[r_{N+1} = \tilde{h}|y_{N+1} = h, \mathcal{R}^l_N] \times \Pr[y_{N+1} = h|\mathcal{R}^l_N] \\ &= \Pr[y_{N+1} = h|\mathcal{R}^l_N],\end{aligned}$$

where $\Pr[y_{N+1} = h|\mathcal{R}^l_N]$ is derived in (39).

For $n \in \{1, 2, \cdots, N\}$, the numerator $\Pr[y_n = h, r_{N+1} = \tilde{h}|\mathcal{R}^l_N]$ can be rewritten as

$$\Pr[y_n = h, r_{N+1} = \tilde{h}|\mathcal{R}^l_N] = \Pr[r_{N+1} = \tilde{h}|y_n = h, \mathcal{R}^l_N] \times \Pr[y_n = h|\mathcal{R}^l_N].$$

Here, $\Pr[y_n = h|\mathcal{R}_N^l]$ is known from the supposition. Now we only need to check if $\Pr[r_{N+1} = \tilde{h}|y_n = h, \mathcal{R}_N^l]$ can be calculated. I rewrite

$$\Pr[r_{N+1} = \tilde{h}|y_n = h, \mathcal{R}_N^l] = \Theta + \Lambda,$$

where

$$\begin{aligned}\Theta =& \Pr[r_{N+1} = \tilde{h}, y_{N+1} = h|y_n = h, \mathcal{R}_N^l] \\ =& \Pr[r_{N+1} = \tilde{h}|y_{N+1} = h, y_n = h, \mathcal{R}_N^l] \times \Pr[y_{N+1} = h|y_n = h, \mathcal{R}_N^l] \\ =& 1 \times \Pr[y_{N+1} = h|y_n = h, \mathcal{R}_N^l] \\ =& \Pr[y_{N+1} = h|y_n = h, \mathcal{R}_N^l],\end{aligned} \tag{41}$$

$$\begin{aligned}\Lambda =& \Pr[r_{N+1} = \tilde{h}, y_{N+1} = l|y_n = h, \mathcal{R}_N^l] \\ =& \Pr[r_{N+1} = \tilde{h}|y_{N+1} = l, y_n = h, \mathcal{R}_N^l] \times \Pr[y_{N+1} = l|y_n = h, \mathcal{R}_N^l] \\ =& x\left[1 - \Pr[y_{N+1} = h|y_n = h, \mathcal{R}_N^l]\right] \\ =& x[1 - \Theta].\end{aligned} \tag{42}$$

If $n = N$, it is straightforward to determine that

$$\Pr[y_{N+1} = h|y_n = h, \mathcal{R}_N^l] = \pi_{hh}.$$

Now let us consider $\Pr[y_{N+1} = h|y_n = h, \mathcal{R}_N^l]$ if $n < N$. Because actual earnings y follow a Markov process, all the past information is fully summarized in the most recent realization, and the prior realizations are informationally irrelevant. Thus,

$$\begin{aligned}\Pr[y_{N+1} = h|y_n = h, \mathcal{R}_N^l] &= \Pr[y_{N+1} = h|y_n = h, \bar{y} = l, r_1 = \tilde{h}, \cdots, r_N = \tilde{h}], \\ &= \Pr[y_{N+1} = h|\bar{y} = h, r_{n+1} = \tilde{h}, \cdots, r_N = \tilde{h}]\end{aligned}$$

and

$$\Pr[y_{N+1} = h|\bar{y} = h, r_{n+1} = \tilde{h}, \cdots, r_N = \tilde{h}] = \Pr[y_{N-n+1}|\bar{y} = h, r_1 = \tilde{h}, \cdots, r_{N-n} = \tilde{h}].$$

Recall that $\mathcal{R}_{N-n}^h \equiv \{\bar{y} = h, r_1 = \tilde{h}, \cdots, r_{N-n} = \tilde{h}\}$. Therefore,

$$\Pr[y_{N+1} = h|y_n = h, \mathcal{R}_N^l] = \begin{cases}\Pr[y_{N-n+1} = h|\mathcal{R}_{N-n}^h] & \text{if } n < N, \\ \pi_{hh} & \text{if } n = N.\end{cases} \tag{43}$$

and

$$
\begin{aligned}
\Pr[y_{N-n+1} = h|\mathcal{R}^h_{N-n}] =& \Pr[y_{N-n+1} = h, y_{N-n} = h|\mathcal{R}^h_{N-n}] + \Pr[y_{N-n+1} = h, y_{N-n} = l|\mathcal{R}^h_{N-n}] \\
=& \Pr[y_{N-n+1} = h|y_{N-n} = h, \mathcal{R}^h_{N-n}] \times \Pr[y_{N-n} = h|\mathcal{R}^h_{N-n}] \\
&+ \Pr[y_{N-n+1} = h|y_{N-n} = l, \mathcal{R}^h_{N-n}] \times \Pr[y_{N-n} = l|\mathcal{R}^h_{N-n}] \\
=& \pi_{hh} \Pr[y_{N-n} = h|\mathcal{R}^h_{N-n}] + \pi_{lh} \left[1 - \Pr[y_{N-n} = h|\mathcal{R}^h_{N-n}]\right],
\end{aligned}
$$

where $\Pr[y_{N-n} = h|\mathcal{R}^h_{N-n}]$ is known from the supposition, since $N - n < N$. Therefore, Θ and Λ can be both calculated. Hence, the numerator in (38) can be derived following this procedure. The numerator is obtained

$$
\begin{aligned}
&\Pr[y_n = h, r_{N+1} = \tilde{h}|\mathcal{R}^l_N] = \\
&\begin{cases}
\pi_{hh} \Pr[y_N = h|\mathcal{R}^l_N] + \pi_{lh} \left[1 - \Pr[y_N = h|\mathcal{R}^l_N]\right] & \text{if } n = N+1, \\
\Pr[y_N = h|\mathcal{R}^l_N] \left[\pi_{hh} + x(1 - \pi_{hh})\right] & \text{if } n = N, \\
\Pr[y_n = h|\mathcal{R}^l_N] \Big\{ \pi_{hh} \Pr[y_{N-n} = h|\mathcal{R}^h_{N-n}] + & \text{if } n < N. \\
\pi_{lh} \left[1 - \Pr[y_{N-n} = h|\mathcal{R}^h_{N-n}]\right] & \\
+x\{1 - \pi_{hh} \Pr[y_{N-n} = h|\mathcal{R}^h_{N-n}] - \pi_{lh} \left[1 - \Pr[y_{N-n} = h|\mathcal{R}^h_{N-n}]\right]\} \Big\} &
\end{cases}
\end{aligned}
\tag{44}
$$

Now combining the expressions (40) and (44), it has been shown that $\Pr[y_n = h|\mathcal{R}^l_N, r_{N+1} = \tilde{h}]$ can be calculated using $\Pr[y_n = h|\mathcal{R}^l_N, r_N = \tilde{h}]$. The same procedure can be repeated for $\Pr[y_n = h|\mathcal{R}^h_N, r_{N+1} = \tilde{h}]$ as follows.

$$
\Pr[y_n = h|\mathcal{R}^h_N, r_{N+1} = \tilde{h}] = \frac{\Pr[y_n = h, r_{N+1} = \tilde{h}|\mathcal{R}^h_N]}{\Pr[r_{N+1} = \tilde{h}|\mathcal{R}^h_N]}.
$$

where the denominator is

$$
\begin{aligned}
\Pr[r_{N+1} = \tilde{h}|\mathcal{R}^h_N] =& \pi_{hh} \Pr[y_N = h|\mathcal{R}^h_N] + \pi_{lh} \left[1 - \Pr[y_N = h|\mathcal{R}^h_N]\right] \\
&+ x\{1 - \pi_{hh} \Pr[y_N = h|\mathcal{R}
\end{aligned}
$$

[redacted]

and the numerator is

$$\Pr[y_n = h, r_{N+1} = \tilde{h}|\mathcal{R}_N^h] =$$
$$\begin{cases} \pi_{hh}\Pr[y_N = h|\mathcal{R}_N^h] + \pi_{lh}\left[1 - \Pr[y_N = h|\mathcal{R}_N^h]\right] & \text{if } n = N+1, \\ \Pr[y_N = h|\mathcal{R}_N^h]\left[\pi_{hh} + x(1-\pi_{hh})\right] & \text{if } n = N, \\ \Pr[y_n = h|\mathcal{R}_N^h]\Big\{\pi_{hh}\Pr[y_{N-n} = h|\mathcal{R}_{N-n}^h] + & \text{if } n < N. \\ \pi_{lh}\left[1 - \Pr[y_{N-n} = h|\mathcal{R}_{N-n}^h]\right] & \\ +x\{1 - \pi_{hh}\Pr[y_{N-n} = h|\mathcal{R}_{N-n}^h] - \pi_{lh}\left[1 - \Pr[y_{N-n} = h|\mathcal{R}_{N-n}^h]\right]\}\Big\} & \end{cases}$$