A theory of mergers and merger waves[*]

Bingyong Zheng[†]

July 4, 2008

Abstract

We consider a model of horizontal mergers in a market consisting of Cournot firms pro-

era came to an end. After a short period of repose, nevertheless, M&A activities reached another high in 2006; for the first time, the announced value of M&A reached $4 trillion in one year.[1] The trend continued in mid-2007 until the subprime loan crisis in U.S. put an end to this wave. Yet, the value of M&A in the first two quarters of 2007 still reached $2.7 trillion, 58% higher than the first half of 2006.[2] In this paper, we provide a simple model of horizontal mergers to explain merger waves and related stylized facts from the empirical literature.

This literature (e.g., Mitchell and Mulherin 1996, Andrade et al. 2001) has uncovered several important facts about mergers and acquisitions over the 20th century. The first stylized fact is the wave phenomenon mentioned above. Historians and M&A specialists have identified five merger waves in the United States, not including the most recent flurry of M&A in 2006 to 2007. These periods of intense merger activity are: 1897–1907; 1916–1929; 1965–1969; 1981–1989; and 1993–2000.

The second stylized fact identified by empirical studies is that mergers concentrate in industries that are subject to exogenous shocks. These shocks include: technological innovations; supply shocks or changed market demand; and deregulation. The third stylized fact is that acquirers' returns are, on average, negative (see Bradley et al. 1988, Jennings and Mazzeo 1991, Banerjee and Owers 1992, Byrd and Kickman 1992).[3]

These stylized facts are puzzling and hard to reconcile theoretically. Currently, there are two

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[1] The Economist, *Economic and financial indicators*, Jan 23, 2007.

[2] The Economist, *Economic and financial indicators*, Jul 7, 2007.

[3] The statistical evidence on whether mergers create value for shareholders is based primarily on short-window event studies. There are two frequently used event windows. The shorter one is from one day before to one day after the announcement of merger, and the longer window begins several days before the announcement and ends at the close of the merger. In a recent work, Andrade et al. (2001) have argued that the empirical finding of average negative return could be due to negative stock market reaction to equity issue for e20.3054(v)0.34(m)4552(s)-0.3528.23893(d)-226.

approaches to explain the empirical findings: the behaviora

the least cost way to alter industry structure. Hence, mergers are an efficient response to regime shifts. However, these neoclassical theories face difficulties reconciling the findings of negative acquirers return. Further, they offer no foundation for the presumption that merger is an efficient response to an exogenous shock. Hence, a theoretical model is needed to explain how profit-maximizing firms react to exogenous shocks through merger decisions.

This paper is an attempt to fulfill this task. The model developed here is a standard Cournot model with homogeneous products and quadratic cost function. We believe the focus on horizontal mergers is without loss of generality, as horizontal mergers are the most common in practice. For example, all merger waves in the 20th century except the conglomerate merger wave in 1965–1969 were primarily mergers of firms in the same market in attempt to increase scale. In our model, the market initially consists of N identical firms with quadratic cost function. In the pre-merger market, firms engage in standard Cournot competition. In facrof exogenous shocks, such as supply shock, changed market demand or a more friendly antitrust policy toward mergers, firms react optimally through merging and acruisitions to increase operation scale and market power. After merger, firms, both merged and non-merged ones, engage in Cournot competition and maximize profit. In the post-merger equilibrium, the merged firm will be different from non-merged firms, as it is "larger" in the sense that it combines the production capacrty of member firms. As a result, the merged firm necessarily enjoys lower production cost than an outside firm at any given output.

Even this simple model brings forth several interesting results. The first prediction is on profitability of mergers. There is an incentive to merge when the cost saving advantage is large or the slope of inverse market demand is low. This is different from previous researches on horizontal mergers, for example, Salant, Switzer, and Reynolds (1983). In a standard Cournot setting, Salant et al. have showed that merger is generally unprofitable unless more than 80% of firms merged into one.[4]

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[4]When products are differentiated, Deneckere and Davidson (1985) have shown that any merger is profitable when firms engage in price competition. A recent work by Banal-Estãnol (2005) shows that in an uncertain environment when firms have independent private information, the information-sharing advantage may increase firm's incentives to merge.

The second prediction is on the pattern of merger activities. The model predicts that, in the absence of entry, mergers and acquisitions always take place in waves. Whenever it is profitable for the first merger, subsequent mergers becomes increasingly profitable; and waves of mergers will take place. In addition, our model predicts that some mergers may take place even if they are not profitable by themselves. Here, what the initially unprofitable mergers have accomplished is simply to get the bandwagon of mergers started. As the wave of mergers move on, ultimately, all mergers become profitable. Hence, while retaining the assumptions of market efficiency and profit-maximization, our model can explain negative returns for acquiring firms as well as merger waves.

The last prediction is that market can be fully monopolized when entry is not allowed and there were no outside intervention from the antitrust authority. This is so as, after the first wave of mergers in which every two oligopolists have merged into a larger oligopolist, they will have further incentives to consolidate in the post-merger market, well until the market is fully monopolized. Therefore, our work also contribute to the debate on effectiveness of antitrust policy and enforcement. In a recent work, Crandall and Winston (2003) have questioned the effectiveness of antitrust policy and whether its enforcement increases competition to the benefit of consumers. They argue that the antitrust policy is ineffective as the antitrust authorities have difficulties sorting out mergers that may stifle competition. Baker (2003), on the other hand, argues that it is hard to gauge the gain from antitrust enforcement as its primary benefit comes from the deterrence on anticompetitive conduct that is never observed. Our model provides support to Baker on the deterring effect of antitrust policy and enforcement. In intervening merger cases that may signific e   merg

a model on the possible mechanism that makes merger an efficient response to the exogenous shocks for profit-maximizing firms. Thus, it contributes to our understanding of mergers and merger waves, and is an useful addition to the literature.

Our work is related to Perry and Porter (1985) and Kamien and Zang (1991) that show merger could be profitable when firms have a quadratic form cost function. A more closely related work is

We assume $\alpha > 0$, $\beta > 0$ and $A > \beta$. The model is similar to that of Heywood and McGinty (2007), however, unlike in that paper, we put no restrictions on b, as we wish to examine the effect of changes in b on firms' incentives to merge.

With N firms and no mergers, the unique symmetric Cournot equilibrium is as follows. Each firm produces output

$$q^* = \frac{A - \beta}{b(N+1) + 2\alpha},$$

and the market price is given by

$$P^* = \frac{bA + 2\alpha A + bN\beta}{b(N+1) + 2\alpha}.$$

Each firm's profit in equilibrium is

$$\pi^0(N) = \frac{(A - \beta)^2(b + \alpha)}{[b(N+1) + 2\alpha]^2} - \theta.$$

Next, we look at the Cournot equilibrium when the market becomes asymmetric due to a single merger. A merger of m firms results in an equilibrium with $N - m$ smaller firms and one large firm (denoted Firm C) with m plants. As each plant of Firm C has an identical strictly convex cost function, the optimal allocation of production across its plants is a symmetric one. That is, each of the m plants produces an output of q_C/m, where q_C denotes the total output of Firm C. Thus, Firm C has the total cost function

$$C(q_C) = \frac{\alpha}{m}q_C^2 + \beta q_C + \theta_c(m).$$

Here $\theta_C(m)$ denotes the post-merger fixed cost of Firm C, which was formed from m smaller firms. Clearly, the value of $\theta_C(m)$ relative to $m\theta$ will influence the value of any merger. As we are interested in the impact of production cost-savings on merger activity, we will concentrate on the case in which there is no fixed cost saving, so $\theta_C(m) = m\theta$. We characterize the circumstances under which merger is profitable in this restricted situation, while keeping in mind that allowing fixed-cost saving will increase incentives to merge.

Let $n = N - m$ be the number of outside firms (also referred to as non-merged firms) who haven't merged with any other firm. Let $\pi_C^1(m)$ (respectively, $\pi_{ni}^1(m)$) denote the profit of Firm

C (respectively, a non-merged firm), in an equilibrium with this single merger of m firms. When there is no risk of confusion, we may simply use π_C^1 and π_n^1. After an m firm merger, Firm C maximizes profit π_C^1 by choosing the output q_C, where

$$\pi_C^1 = \left[A - b \left(\sum_{j=1}^{n} q_j + q_C \right) \right] q_C - \left[\frac{\alpha}{m} q_C^2 + \beta q_C + \theta_C \right].$$

Each outside firm i maximizes profit π_n^1 by choosing output q_i, where

$$\pi_n^1 = \left[A - b \left(\sum_{j=1}^{n} q_j + q_C \right) \right] q_i - \left[\alpha q_i^2 + \beta q_i + \theta \right]$$

Solving for the Cournot equilibrium gives output quantities of:

$$q_C = \frac{(A - \beta)(2\alpha + b)}{nb^2 + 2b^2 + 4\alpha b + \frac{2\alpha}{m}(bn + b + 2\alpha)}, \tag{1}$$

$$q_i = \frac{(A - \beta)(\frac{2\alpha}{m} + b)}{[nb^2 + 2b^2 + 4\alpha b + \frac{2\alpha}{m}(bn + b + 2\alpha)]},$$

and an equilibrium price of:

$$P = \frac{(2\alpha + b)(Ab + \beta b + \frac{2A\alpha}{m}) + n\beta b^2 + \frac{2n\alpha\beta b}{m}}{[nb^2 + 2b^2 + 4\alpha b + \frac{2\alpha}{m}(bn + b + 2\alpha)]}.$$

Plugging the price and output into the profit function, we get the profit for Firm C as

$$\pi_C^1 = \frac{(A - \beta)^2(2\alpha + b)(b^2 + 2\alpha b + \frac{\alpha b}{m} + \frac{2\alpha^2}{m})}{[nb^2 + 2b^2 + 4\alpha b + \frac{2\alpha}{m}(bn + b + 2\alpha)]^2} - \theta_C, \tag{2}$$

and the profit for an outside firms as

$$\pi_i^1 = \frac{(A - \beta)^2(\frac{2\alpha}{m} + b)(b^2 + \alpha b + \frac{2\alpha b}{m} + \frac{2\alpha^2}{m})}{[nb^2 + 2b^2 + 4\alpha b + \frac{2\alpha}{m}(bn + b + 2\alpha)]^2} - \theta. \tag{3}$$

It is worth pointing out that when $m = 1$ and $n = N - 1$, the two profit functions are identical and are the same as the pre-merger equilibrium profit function,

$$\pi_C^0 = \pi_n^0 = \pi^0(N) = \frac{(A - \beta)^2(b + \alpha)}{[b(N + 1) + 2\alpha]^2} - \theta.$$

To determine whether there is an incentive for m firms to merge, we examine the difference in total profits for an m-plant firm before and after the merger, $D^1(m) = \pi_C^1 - m\pi^0$, where π^0 is the profit for one firm in the pre-merger Cournot equilibrium. That is,

$$D^1(m) = \frac{(A-\beta)^2(2\alpha+b)(b^2+2\alpha b + \frac{\alpha b}{m} + \frac{2\alpha^2}{m})}{[nb^2+2b^2+4\alpha b + \frac{2\alpha}{m}(bn+b+2\alpha)]^2} - \frac{m(A-\beta)^2(b+\alpha)}{[b(N+1)+2\alpha]^2} + m\theta - \theta_C.$$

When $D^1(m) > 0$, total profit for the merged firm is greater than total profit of member firms before merger, and there is an incentive to merge. Otherwise, it is not profitable to merge.

3 Two-firm mergers

3.1 First merger

When $m = 2$, the profit difference is:

$$D^1(2) = \frac{(A-\beta)^2(2\alpha+b)(b^2+\frac{5\alpha b}{2}+\alpha^2)}{[Nb^2+N\alpha b+3\alpha b+2\alpha^2]^2} - \frac{2(A-\beta)^2(b+\alpha)}{[b(N+1)+2\alpha]^2} + 2\theta - \theta_C$$

As noted above, we assume there is no fixed-cost saving, so $2\theta - \theta_C = 0$. If, in addition, firms had a constant marginal cost, so that $\alpha = 0$, the profit difference would be

$$D^1(2) = \frac{(A-\beta)^2}{b}\left[\frac{1}{N^2} - \frac{2}{(N+1)^2}\right].$$

Then $D^1(2) > 0$ requires that $N < \frac{1}{\sqrt[2]{2}-1} = 2.4142$, so there would no incentives to merge unless, there were only two firms in the market initially and a merger would result in full monopolization.

However, once increasing marginal costs are allowed for, we have the following result, which is proved in the Appendix:

Proposition 1. *For any market size, N, there is a value $\varepsilon_2(N)$ such that a 2-firm merger is profitable if and only if*

$$\frac{\alpha}{b} > \varepsilon_2(N).$$

Hence, firms' incentives to merge depend on two factors: the quadratic cost parameter α and the slope of the inverse market demand curve b. It is not surprising that the cost parameter

α affects the incentives to merge, as one would expect cost-savings to increase the profitability of mergers, and α determines how fast plant-level costs rise with output, and hence how much cost-saving there is from being able to spread output over several plants. It is less obvious how the slope of market demand curve plays a role, but the intuition turns out to be fairly simple. A merged firm can increase A

increase. This, however, gives non-merged firms an incentive to expand output and profit from the output reduction by the merged firm. Thus, mergers that would increase total industry profits may not occur due to this "free-rider" problem. Kamien and Zang (1990) have made a similar point in their decentralized merger model.

3.2   Subsequent mergers and merger wave

In above, we have showed the profitability of the first two-firm merger depends on production cost-savings and the slope of inverse market demand. This raises the question of whether further mergers are profitable, given that an initial two-firm merger is profitable. One obvious way to pursue this question is to ask: If the market has already seen $h \geq 1$ two-firm mergers occur, does it imply that a further merger by two previously independent firms is profitable?

The Cournot equilibrium output for an m-plant firm and a single-plant firm can then be derived as:

$$q_C^h = \frac{(A-\beta)(2\alpha+b)}{[(n^h+h+1)b^2+2(h+1)\alpha b+\frac{2(n^h+1)\alpha b}{m}+\frac{4\alpha^2}{m}]} \tag{5}$$

$$q_n^h = \frac{(A-\beta)(\frac{2\alpha}{m}+b)}{[(n^h+h+1)b^2+2(h+1)\alpha b+\frac{2(n^h+1)\alpha b}{m}+\frac{4\alpha^2}{m}]}$$

The equilibrium market price is:

$$P^h = \frac{(b+2\alpha)(Ab+\frac{2A\alpha}{m}+h\beta b)+\frac{2n\alpha\beta b}{m}+n^h\beta b^2}{[(n^h+h+1)b^2+2(h+1)\alpha b+\frac{2(n^h+1)\alpha b}{m}+\frac{4\alpha^2}{m}]}.$$

In the asymmetric Cournot equilibrium, the profit for an *m*-plant firm is

$$\pi_C^h = \frac{(A-\beta)^2(2\alpha+b)(b^2+2\alpha b+\frac{\alpha b}{m}+\frac{2\alpha^2}{m})}{[(n^h+h+1)b^2+2(h+1)\alpha b+\frac{2(n^h+1)\alpha b}{m}+\frac{4\alpha^2}{m}]^2} - \theta_C,$$

while the profit for a single-plant firm is

$$\pi_n^h = \frac{(A-\beta)^2(\frac{2\alpha}{m}+b)(b^2+\alpha b+\frac{2\alpha b}{m}+\frac{2\alpha^2}{m})}{[(n^h+h+1)b^2+2(h+1)\alpha b+\frac{2(n^h+1)\alpha b}{m}+\frac{4\alpha^2}{m}]^2} - \theta.$$

Note that when $h=1$, $n^h=N-m=n$, π_C^h equals π_C^1 as in (3) and π_n^h equals π_n^1 as in (2).

We now focus our attention on the case of $m=2$. That is, we examine the incentives for any two remaining non-merged firms to merge when there have already been h previous 2-firm mergers. Note that when $m=2$, $n^h=N-2h$ and $n^{h+1}=N-2(h+1)$.

Before determining the profitability of mergers, it is instructive to look at the effect of additional mergers on firms' output and market price in equilibrium. For any $h \geq 0$, n^h decreases in h. In this case,

$$q_C^h = \frac{(A-\beta)(2\alpha+b)}{[(N-h+1)b^2+(N+3)\alpha b+2\alpha^2]},$$

$$q_n^h = \frac{(A-\beta)(\alpha+b)}{[(N-h+1)b^2+(N+3)\alpha b+2\alpha^2]},$$

$$P^h = \frac{(2\alpha+b)(Ab+A\alpha+h\beta b)+(N-2h)(\alpha\beta b+\beta b^2)}{[(N-h+1)b^2+(N+3)\alpha b+2\alpha^2]}.$$

Clearly, both a merged firm's output q_C^h and a non-merged firm's output q_n^h are increasing in h. Further, we have that:

$$\frac{\partial P^h}{\partial h} = \frac{b^2(2\alpha+b)(\alpha+b)(A-\beta)}{[(N-h+1)b^2+(N+3)\alpha b+2\alpha^2]^2} > 0 \qquad (6)$$

due to our assumption that $A > \beta$.

Now denote the profit difference $\pi_C^{h+1} - 2\pi_n^h$ by D^{h+1}, so that

$$D^{h+1} = \frac{(A-\beta)^2(2\alpha+b)(b^2+2.5\alpha b+\alpha^2)}{[(N-h)b^2+(N+3)\alpha b+2\alpha^2]^2} - \frac{2(A-\beta)^2(\alpha+b)^3}{[(N-h+1)b^2+(N+3)\alpha b+2\alpha^2]^2}.$$

Simplifying the profit difference equation, we get

$$\begin{aligned}
D^{h+1} &= \{(-1.5N^2-3N+2h+30.5)\alpha^3 b^4 + 2\alpha^5 b^2 + (-2N+14)\alpha^4 b^3 \\
&\quad + (-4N^2+3Nh+2N+h+28)\alpha^2 b^5 + (-3.5N^2+5Nh+5N-1.5h^2 \\
&\quad - 3h+10.5)\alpha b^6 + [-(N-h)^2+2(N-h)+1]b^7\}\Psi(h),
\end{aligned}$$

where

$$\Psi(h) = \frac{(A-\beta)^2}{[(N-h-1)b^2+(N+3)\alpha b+2\alpha^2]^2[(N-h+2)b^2+(N+3)\alpha b+2\alpha^2]^2}.$$

To simplify notation, we also denote the term in brackets $\{\}$ in D^{h+1} that multiplies $\Psi(h)$ as $\Gamma(h)$ and thus, $D^{h+1} = \Gamma(h)\Psi(h)$. Clearly, $\Psi(h)$ is strictly positive for any $h \in \{0,\dots,N/2-1\}$, so $D^{h+1} > 0$ if an only if $\Gamma(h) > 0$.

Proposition 1 shows that when α/b is large enough, there is an incentive to merge, $D^1 > 0$. It turns out this cost saving is also large enough to trigger a sequence of mergers in the symmetric N-firm market.

Proposition 2. *Suppose the first two-firm merger is profitable. Merger is increasingly profitable, that is,*

$$\forall h \geq 1, \quad D^{h+1} > D^h.$$

Therefore, whenever there exists an incentive to merge in the symmetric N-firm market, a sequence of mergers and acquisitions would be triggered by the first merger, with each additional merger makes any further merger even more profitable. This explain why mergers occur in waves. For example, when an industry is subject to exogenous shock, market demand change or supply

shock that affects production costs, in our model, b decreases, or α increases, firms will have an incentive to merge with each other. As each additional merger makes subsequent mergers more profitable, we would expect a wave of merger activities in response to exogenous shocks that changed firms's cost, or market demand.

In addition, our model has another implication, that is, mergers may take place even if a two-firm merger strictly reduce the total profit of merged firms. To see this, it is helpful to treat a wave of mergers as a sequential-move game. Each additional merger makes previous mergers as well as subsequent mergers more profitable. This is true as the reduced number of firms after mergers will reduce competition in the market, pushing up market price and increasing firms' profit.

We consider a market with N firms with each pair of firms separately making merger decisions. Without loss of generality, assume N is an even number. As before, we focus on two-firm mergers. In the sequential-move game of mergers, firms first make merger decisions and then compete in output to realize their profits. Figure 1 shows the timing of events. We denote the $M_1, M_2 \dots, M_{N/2}$ as two-firms mergers in the sequence. For $j = 1, 2, \dots, N/2$, let firms $(2M_j - 1, 2M_j)$ be the two single-plant firms that can choose whether to "merge" or "not" to merge into firm M_j. Firms take turns to make choices, with firms $1, 2$ being the first to move and firms $N - 1, N$ the last to move. Whenever two firms $(2M_j - 1, 2M_j)$ play "not", the game ends, with M_{j-1} merged firms in the market while the rest of firms $2M_j - 1, 2M_j, \dots, N$ remaining as single plant firms. After the merging game ends, firms engage in Cournot competition to maximize profits. For example, if firm $1, 2$ choose not to merge, no merger would take place and the game ends with no mergers and each firm would get a profit of $\pi^0(N)$. On the other hand, if all pairs of firms, firms 1 and 2, ..., and firms $N - 1$ and N choose "merge", the game ends with $N/2$ mergers and each merged firm realizing a profit of $\pi_C^{N/2}$.

We already know that $D^{h+1} = \Gamma(h)\Psi(h)$ and $\Psi(h) > 0$ for all h, and will also show in the appendix that $\Gamma(h)$ is strictly increasing in h. Thus, for some parameters (N, α, b), it may be the case that merger M_j is not profitable, $D^j < 0$, while merger M_k $(k > j)$ is profitable given mergers $1, \dots, M_{k-1}$ have occurred. The extreme case of this scenario is that $D^{N/2} > 0$ (or $\Gamma(N/2 - 1) > 0$), while $D^j < 0$ (or $\Gamma(j-1) < 0$) for $j < N/2$. The following result shows that even in this case,
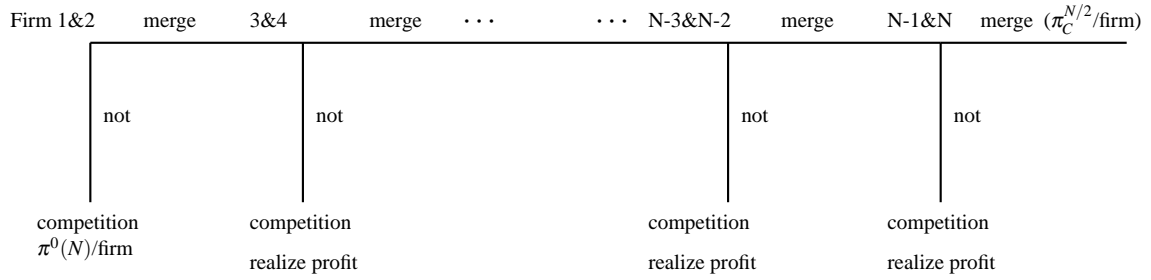
Figure 1: Timing of events

merger wave may still take place.

Proposition 3. *Suppose $D^{N/2} > 0$, all firms play "merge" in the unique subgame perfect Nash equilibrium of the sequential-move game.*

The condition $D^{N/2} > 0$ implies that, conditional on all the other firms in the market have merged, it is profitable for firm $N-1$ and N to play "merge." However, the condition does not state that the two-firm mergers between firm 1 and 2, between firm 3 and 4, etc., are profitable by themselves. Suppose that $D^1 < 0$ but $D^{N/2} > 0$; the first two-firm merger is not profitable, but the last merger would be profitable had all the previous pairs played "merge." In this case, we would expect a sequence of mergers to take place with every pair of firms $(2M_j - 1, 2M_j)$ choose "merge." In fact, as long as the last two-firm merger is profitable, $D^{N/2} > 0$, every pair of firms playing "merge" is the unique subgame perfect Nash equilibrium of the sequential merger game, even if $D^h < 0$ for all $h < N/2$. Hence, mergers and merger wave can take place even if the mergers at the early stage of the wave are not profitable by themselves. This is true as firms understand that, as the wave complete and competition is reduced, all merged firms will get higher total profit than in the pre-merger market. This explains the puzzle that the announcement period abnormal returns to acquirers are negative on average.

Schleifer and Vishny (2003) suggest that merger and acquisitions are driven by stock market valuations. According to them, financial market is inefficient and thus, some firms may be valued incorrectly. Managers of firms understand the market inefficiencies, and take advantage of them through mergers and acquisitions. In their model, acquiring firm may be overvalued by the stock market before the acquisition while the target firm may be undervalued. Another theory proposed by Gorton et al. (2005) suggest that mergers and merger waves can occur when managers prefer to their firms to remain independent rather than acquired. They have assumed that large firms are less likely to be acquired. Therefore, managers may engage in unprofitable defensive acquisitions, thereby reducing the chance of being acquired by another firm.

In contrast to the behavioral theories, none of the existing neoclassical theories can explain the stylized fact of average negative announcement period returns. Therefore, the empirical evidence seems to be in favor of the managerial incentive explanation. However, this need not be so, as negative abnormal return does not have to contradict profit-maximization by firms, as the current model shows. Our model predicts that there are cases in which the exogenous shock is not sufficiently large to make early mergers profitable, but merger wave can still take place. This means that some mergers, especially those at the early stage of the wave, may reduce the merged firms' total profit initially, while at the same time, profit for outside firms strictly increase. This may result in a negative announcement period return for the acquiring firm, as an efficient stock market reacts to the short-run unprofitable mergers. However, as the wave moves on, merged firms' profit gradually increase. Thus, our model also predicts that operation performance would improve over time, whereas the behavior theories predict no such improvement in operating performance post-merger.

This prediction of improved performance over time is consistent with the empirical findings by some recent works on firms' post-merger performance. One hypothesis of these studies is that if mergers truly create value for shareholders, the gains would eventually show up in the firms' operating cash flows. Using samples from the U.S.,

improvements in industry-adjusted, operating cash flows post-takeover from a sample of U.S. mergers. Using a sample of takeovers in the UK over the period 1985–1993, Powell and Stark (2005) report modest improvements in operating performance. Hence, the empirical evidence suggest that mergers and acquisitions do increase firms' performance and are profitable for firms involved in the mergers.

3.3 Full-monopolization

For exposition purpose, we suppose that there are 2^k ($k \in \mathbb{Z}_{++}$) number of firms in the market before any merger has happened, $N = 2^k$. Also suppose there is an incentive to merge in the symmetric pre-merger market, $D^1 > 0$. Proposition 2 indicates that the first merger triggers a sequence of mergers, in which two firms form a two-plant firm. After the first wave of mergers, the market is symmetric with $N/2$ larger firms, each of which now consists of 2 plants and has the cost function

$$C(q_C) = \frac{\alpha}{2} q_C^2 + \beta q_C + \theta_C.$$

What will happen in the post-merger market? Will there be any further incentives to merge? The answer is yes. There is definitely an incentive for further mergers and acquisitions in the market. Without any outside intervention, a second wave of mergers would ensue. Furthermore, new waves of merger will take place after the second wave, well until the market is fully monopolized. This result is summarized as follows.

Proposition 4. *In a Cournot oligopoly market with N firms, where $N = 2^k$, if a two-firm merger is profitable, then $N/2$ round of mergers can take place until the market is fully monopolized.*

Thus, if there ever exists an incentive to merge in a symmetric N-firm market, then, unless blocked by the antitrust authority, the consolidation process could continue till there is only one firm left in the market. Of course, this does not imply that full monopolization will take place in real world, as antitrust authorities can block large mergers in concentrated market.

In the U.S., firms have been required to notify the antitrust authorities of all mergers above a certain size since 1976. The antitrust enforcement agency may require additional informa-

tion concerning the merger for further investigation. In some cases, the antitrust authority can challenge the merger and prevent it from being completed. For example, recently, the Federal Trade Commission has blocked Pepsi's planned acquisition of 7UP and Coca Cola's acquisition of Dr Pepper. In 2001, the U.S. Department of Justice blocked General Dynamics' merger with Newport News Shipbuilding, the only other nuclear submarine builder for the U.S. Navy (Baker 2003). Therefore, there is probably little risk of monopolization with the existence of antitrust policy. However, our analysis does show that one needs to take into account the benefits from deterrence of antitrust law in quantifying the effect of antitrust policy on consumer welfare.

4 Merger and entry

In above we assume that entry is not possible and thus, any mergers necessarily reduces the number of firms and with it competition in the market. This may be appropriate for industries in which there are barriers that make new entry too costly, but may not be realistic for many industries in which entry barrier is not prohibitive. In what follows, we extend the basic model by allowing entry by outside firms into the market under consideration.

To model firms' entry and merger behavior, we assume that there are many but finite potential entrepreneurs who might start a firm in this market. Entrepreneurs may have different abilities operating a firm and this will have effect on firms total cost. We assume that the total costs of any firm i in the market that is operating m plants and producing output q_i are given by:

$$C(q_i) = \frac{\alpha q_i^2}{m} + \beta q_i + \theta_i + Ei.$$

Each entrepreneur gets a draw of E_i from some distribution F with support $[e, \infty)$ where $e > 0$. If we think now about the original Cournot-Nash equilibrium with no mergers but entry, there will be some number of firms, N, that are operating in the market, and these will be operated by the N potential entrepreneurs who got the N lowest draws from the distribution.

All firms in the no merger Cournot equilibrium will earn the same *gross profit* π_i, where

$$\pi_i = \frac{(A - \beta)^2 (b + \alpha)}{[b(N+1) + 2\alpha]^2} - \theta_i.$$

But firms' economic profits Π_i (net of E_i) will vary. Without loss of generality, we number the firms $1, 2, ..., N$ in order of increasing E_i values, then firm N will be the marginal firm, in that it is making just enough to keep its entrepreneur in this industry,

$$E_N = \pi^0(N).$$

Fact 1. *No firm will exit the market after any mergers.*

This is obvious as incumbent firms profits can not decrease in any merger equilibria. First, the merged firm will not exit the market. For two firms to have an incentive to merge, their total profit after the merger must be greater than before the merger and thus, the merged firm should not exit. Second, in the absence of entry, profit for each of the non-merged firms strictly increases after any merger. This implies no incumbent will exit the market after any mergers.

A merger involves one entrepreneur i paying the other entrepreneur j to exit, with the merged firm being operated by the remaining entrepreneur i. We assume that the entrepreneur j who sold his firm can not enter the market again and thus, entry decision only involves entrepreneurs not currently in the market. This assumption is made without loss of generality as in real world, any buying-out contract always includes exclusive clause preventing the entrepreneur from entering the same market in some period of time.

If the N-firm is marginal before the merger,

$$\pi^0(N) = E_N,$$

it turns out that allowing entry will have no effect on the incentive for the first merger.

Proposition 5. *If firm N with E_N has zero profit in the pre-merger equilibrium, then the incentive for the first merger remains unchanged with or without entry.*

Consider a N-firm market with (α, b) and entry barrier due to government regulations. Suppose initially firm N breaks even and (α, b) is such that $\alpha/b > \varepsilon_2(N)$ and thus merger is profitable. Now suppose the government deregulate and the entry barrier is removed, Proposition 5 says that merger remains profitable even when potential entrant might enter the market. In fact, no entry

would occur after the first two-firm merger. The reason is simple. Prior to the merger, the N firms are identical in technology, have same marginal cost at any given output. The firm with the highest E breaks even. If a new firm enters the market after the first two-firm merger, the post-merger post entry market would have N firms with one firm more efficient than the other N-1 firms. This merged firm would produce more than the other N-1 firms. The profit for each of the N-1 firm can not be greater than a firm's profit before any merger and entry. This would imply the firm with the highest E would have a loss in the post-merger post entry equilibrium. Therefore, no entry would occur even if in post-merger market, all firms make positive profits. Proposition 1 still holds, even if entry is allowed.

Of course, this result relies upon the assumption that in the pre-merger equilibrium, firm N is marginal. This may not be true if we allowed the possibility that in the original no-merger equilibrium,

$$\pi^0(N) > E_N \quad \text{but} \quad \pi^0(N+1) < E_{N+1}.$$

In this case, a two-firm merger may induce subsequent entry by firm N+1 into the market. However, the first merger does not induce entry only if E_{N+1} is large, for example, if

$$E_{N+1} \geq \pi^0(N),$$

that is, firm N+1 could not profitably enter the pre-merger market when there were only N-1 firms.

In addition, we know that no entry would occur as long as

$$\frac{(A-\beta)^2(\alpha+b)^3}{[(N+2-h)b^2 + (N+4)\alpha b + 2\alpha^2]^2} - \theta < E_i,$$

where i is the firm with the lowest E among all potential entrant firms. The left-hand side is the expected gross profit (before subtracting E_i) for the entrant firm in a Cournot market with h 2-plant firms and $N + 1 - 2h$ single-plant firm. As long as no entry occurs, there is always an incentive to merge after the first merger. However, once a merger may induce subsequent entry, this reduces firms' incentives to further merge.

Proposition 6. *A merger is unprofitable if it induces subsequent entry.*

Proof. Let h be number of merged firms in the market as defined in Section 3. Now suppose two incumbent firms merge and firm $N+1$ enter the market, thus resulting in $(h+1)$ merged firms and $(N-2h-1)$ non-merged firms, the gross profit for each merged firm would be

$$\tilde{\pi}_n^{h+1} = \frac{(A-\beta)^2(2\alpha+b)(b^2+2.5\alpha b+\alpha^2)}{[(N-h+1)b^2+(N+4)\alpha b+2\alpha^2]^2} - 2\theta.$$

The gross profit for a non-merged firm in a market with h merged firms and $(N-2h)$ non-merged firms is

$$\pi_n^h = \frac{2(A-\beta)^2(\alpha+b)^3}{[(N-h+1)b^2+(N+3)\alpha b+2\alpha^2]^2} - \theta.$$

It is clear that

$$\tilde{\pi}_n^{h+1} < \frac{(A-\beta)^2(2\alpha+b)(b^2+2.5\alpha b+\alpha^2)}{[(N-h+1)b^2+(N+3)\alpha b+2\alpha^2]^2} - 2\theta < 2\pi^h.$$

Hence we conclude that, whenever a merger may induce entry by outside firms, there is no incentives for any incumbent firms to merge. $\qquad\square$

Entry plays the role of damper on mergers. One implication of this result is that partial or complete monopolization may not be possible in market with no or fairly low entry barrier. And the antitrust authority should only worry about anticompetitive mergers in market with severe entry barriers.

5 Conclusion

Empirical research on mergers and acquisitions has found some important stylized facts of merger activity over the last century. First, mergers occur in waves. Second, mergers concentrate in industries for which a regime change can be be identified. Third, the average stock market return to acquiring firms are negative.

Various theories have been advanced to explain these stylized facts. These theories have explained some of the mergers over the last century and thus are relevant to a comprehensive understanding of what drives mergers and acquisitions. However, none seems being able to

reconcile all stylized facts. Therefore, there is room for an alternative model as we present in this paper.

We consider a model of horizontal mergers between Cournot firm

Now, note that the LHS is increasing in $\varepsilon \equiv \alpha/b$, while the RHS is decreasing in ε. Thus, for any value of N, it is possible to find a value of ε large enough that the inequality holds. $\square$

Proof of Proposition 2. To prove the proposition, we show that D^{h+1} is strictly increasing in h for $h \in \{0, 1, \ldots, N/2\}$. Though h takes only discrete

As both $\Psi(0) > 0$ and $\Gamma(0) > 0$ while $\Psi(h)$ and $\Gamma(h)$ increase in h, it follows immediately that for $h > 0$,

$$\Psi(h) > 0, \qquad \Gamma(h) > 0.$$

This condition, together with the conditions that $d\Gamma(h)/dh > 0$ and $d\Psi(h)/dh > 0$, indicates that

$$\frac{dD^{h+1}}{dh} = \Gamma(h)\frac{d\Psi(h)}{dh} + \frac{d\Gamma(h)}{dh}\Psi(h) > 0,$$

and $D^{h+1} > D^h$ for $h \in \{0, 1, \ldots, N/2 - 1\}$. $\qquad\square$

Proof of Proposition 3. To prove this result, we note that the profit for a merged firm π_C^h is strictly increasing in h. After the last merger has been completed, all merges become profitable,

$$\pi_C^{N/2} - 2\pi^0(N) > \pi_C^{N/2} - 2\pi_n^1 > \ldots > \pi_C^{N/2} - 2\pi_n^{N/2-1} = D^{N/2} > 0,$$

even if the merger is not profitable at the time when it took place.

Applying backward induction, it immediately follows that each pair of firms play "merge" when it is their turn to play the game. Hence, we conclude that as long as $D^{N/2} > 0$, a wave of mergers will take place, even if D^h may be negative for $h < N/2$. $\qquad\square$

Proof of Proposition 4. Note that in a market with N firms, the first two-firm merger will be profitable if and only if

$$(-1.5N^2 + N + 8.5)\alpha b^4 + (10 - 2N)\alpha^2 b^3 + 2\alpha^3 b^2 + (-N^2 + 2N + 1)b^5 > 0. \qquad (\text{A}.3)$$

To show full-monopolization is possible under this condition, we only need to show that for a market with N/m firms, where $m \in \{1, 2, 4, \ldots, N/2\}$ and each firm has cost function

$$C = \frac{\alpha}{m}q^2 + \beta q + \theta,$$

the first two-firm merger is profitable. Hence, what we need to show is that (A.3) implies

$$(-1.5N'^2 + N' + 8.5)\alpha'^4 + (10 - 2N')\alpha'^2 b^3 + 2\alpha'^3 b^2 + (-N'^2 + 2N'^5] > 0 \qquad (\text{A}.4)$$

where $N' = N/m$ and $\alpha' = \alpha/m$ for $m \in [2, 4, \ldots, N/2]$.

When $m = 2$, the condition in (A.4) becomes

$$\frac{1}{8}\left[(-1.5N^2 + 2N + 34)\alpha b^4 + (20 - 2N)\alpha^2 b^3 + 2\alpha^3 b^2 + (-2N^2 + 8N + 8)b^5]\right]$$

$$= \frac{1}{8}\left[(-1.5N^2 + N + 8.5)\alpha b^4 + (10 - 2N)\alpha^2 b^3 + 2\alpha^3 b^2 + (-N^2 + 2N + 1)b^5 + \right.$$

$$+ (N + 25.5)\alpha b^4 + 10\alpha^2 b^3 + (-N^2 + 6N + 7)b^5\right]$$

For the discussion to be meaningful, there should be at least 4 firms in the market, $N \geq 4$. In this case,

$$(-1.5N^2 + 2N + 8.5) < (-N^2 + 2N + 1) < 0, \quad (10 - 2N) \leq 2.$$

Clearly, (A.3) implies that $\alpha > b$. When $N = 4$, $(-N^2 + 6N + 7) > 0$ and thus, (A.3) implies (A.4). When $N > 4$, (A.3) would also implies that

$$2\alpha^3 b + (-1.5N^2 + N + 8.5)\alpha b^4 > 0.$$

But

$$(N + 25.5)\alpha b^4 + 10\alpha^2 b^3 + (-N^2 + 6N + 7)b^5$$

$$> (N + 25.5)\alpha b^4 + 8\alpha^2 b^3 + \frac{b}{\alpha}[2\alpha^3 b^2 + (-1.5N^2 + N + 8.5)\alpha b^4]$$

$$> 0$$

Consequently, when $N > 4$, (A.3) implies that (A.4) holds. Thus we can conclude that if the first two-firm merger is profitable in market with N firms, the subsequent two-firm merger between merged firm with $m \geq 2$ plants is also profitable. □

Proof of Proposition 5. When firm N is marginal, it breaks even in the pre-merger Cournot equilibrium,

$$E_N = \pi^0(N) = \frac{(A - \beta)^2(b + \alpha)}{[b(N + 1) + 2\alpha]^2} + 2\theta_i - \theta_i.$$

Now suppose firm N+1 enters the market, gross profit for each of the non-merged firm would be

$$\pi^1(N) = \frac{(A - \beta)^2(b + \alpha)^3}{[(N + 1)b^2 + (N + 4)\alpha b + 2\alpha^2]^2} - \theta_i < \Pi^0(N).$$

This implies that $E_{N+1} > \pi^1(N)$ and firm N+1 suffers a loss by entering the market. □

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