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Regime Shifts, Risk Premiums in the Term Structure, and the Business Cycle

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Recent evidence indicates that using multiple forward rates sharply predicts future excess returns on U.S. Treasury Bonds, with the R^2's being around 30%. The projection coefficients in these regressions exhibit a distinct pattern that relates to the maturity of the forward rate. These dimensions of the data, in conjunction with the transition dynamics of bond yields, offer a serious challenge to term structure models. In this article we show that a regime-shifting term structure model can empirically account for these challenging data features. Alternative models, such as affine specification, fail to account for these important features. We find that regimes in the model are intimately related to bond risk premia and real business cycles.

KEY WORDS: Business cycle; Efficient method of moments; Expectation hypothesis; Regime shifting; Term structure of interest rate.

1. INTRODUCTION

Term structure models with regime shifts, considered by Naik and Lee (1997) and Bansal and Zhou (2002), capture the important feature that the aggregate economy is subject to discrete and persistent changes in the business cycle. The business cycle fluctuations, together with the monetary policy response to them, have significant impacts not only on the short-term interest rate, but also on the entire term structure. Regime-shifting term structure models represent a parsimonious way of introducing interactions between the business cycles, the term structure, and risk premia on bonds. Using the U.S. Treasury yield data from 1964 to 1995, Bansal and Zhou (2002) found that the model-implied regime changes usually lead or coincide with economic recessions. Therefore, the term structure regimes seem to confirm and complement the real business cycles. This evidence also allows for the possibility that this class of term structure models may be able to capture the dynamics of risk premia on bonds.

The most common strategy for understanding bond risk premiums is to study deviations from the expectations hypothesis. One form of the violation, that the regression of yield changes on yield spreads produces negative slope coefficient instead of unity (Campbell and Shiller 1991), has been addressed in many recent articles (e.g., Roberds and Whiteman 1999; Dai and Singleton 2002; Bansal and Zhou 2002; Evans 2003). Another form of violation of the expectations hypothesis is that the forward rate can predict the excess bond return (Fama and Bliss 1987). More recently, Cochrane and Piazzesi (2002) documented that using multiple forward rates to predict bond excess returns generates very high predictability of bond excess returns, with adjusted R^2's from the regression of around 30%. Further, they showed that the coefficients of multiple forward-rate regressors form a tent-shaped pattern related to the maturity of the forward rate. The size of the predictability and nature of projection coefficients is quite puzzling and constitutes a chal-

The main contribution of this article is to account for the predictability evidence from the perspective of latent factor term structure models. When evaluating the plausibility of various term structure models, it is important to not focus exclusively on the predictability issue; previous work (e.g., Dai and Singleton 2000; Bansal and Zhou 2002; Ahn, Dittmar, and Gallant 2002) highlights the difficulties that many received models have in capturing the transition dynamics of yields (i.e., conditional volatility and conditional cross-correlation across yields). The predictability evidence, in conjunction with the transition dynamics, constitutes a sufficiently rich set of data features for discriminating across alternative term structure models and to evaluate their plausibility. The main empirical finding of this article is that the regime-shifting term structure models can simultaneously justify the size and nature of bond return predictability and the transition dynamics of yields. More specifically, we find that models with regime shifts can reproduce the high predictability and the tent-shaped regression coefficients documented by Cochrane and Piazzesi (2002). Additionally, the regime-shifting term structure model reproduces the dynamics of conditional volatility and cross-correlation across yields. In contrast, commonly used multifactor Cox–Ingersoll–Ross (CIR) (Cox, Ingersoll, and Ross 1985) and affine models cannot capture these dimensions of the data. Our overall evidence indicates that incorporating regime shifts is important for interpreting key aspects of Treasury bond market data.

We use U.S. Treasury yield data from 1964–2001. The period 1996–2000 poses a tough challenge for standard asset pricing models, with unprecedented long economic growth and bull market run. At the same time, this period includes several economic recessions and periods of economic boom.

Using the whole sample, we find that the conditional correlation between the long and short yields vary over a range of about 40–80%. The conditional volatilities of the long and short yields also reveal very large variations. Despite this, when evaluating the U.S. Treasury yields data from 1964–2001, our regime-shifting model still stands out as the best-performing candidate. The regime indicator is related to business cycles in the data; for example, the model-based regime indicator predicts the 2001–2002 recession.

To estimate various models under consideration, we use the efficient method of moments (EMM), developed by Bansal, [redacted]

In this section we review the term structure model with regime shifts proposed by Bansal and Zhou (2002). The derivation focuses on a single-factor specification; the multifactor extension is straightforward (see Bansal and Zhou 2002). To capture the idea that the aggregate economy is subject to regime shifts, we model the regime-shifting process as a two-state Markov process, as was done by Hamilton (1989). Suppose that the evolution of tomorrow's regime, $s_{t+1} = 0, 1$, given today's regime, $s_t = 0, 1$, is governed by the transitional probability matrix of a Markov chain,

$$\Pi = \begin{bmatrix} \pi_{00} & \pi_{01} \\ \pi_{10} & \pi_{11} \end{bmatrix}, \qquad (1)$$

where $\sum_{j=0,1} \pi_{ij} = 1$ and $0 < \pi_{ij} < 1$. In addition to the discrete regime shifts, the economy is also affected by a continuous-state variable,

$$X_{t+1} - X_t = \kappa_{s_{t+1}}(\theta_{s_{t+1}} - X_t) + \sigma_{s_{t+1}}\sqrt{X_t}u_{t+1}, \qquad (2)$$

where $\kappa_{s_{t+1}}$, $\theta_{s_{t+1}}$, and $\sigma_{s_{t+1}}$ are the regime-dependent mean reversion, long-run mean, and volatility parameters. All of these parameters are subject to discrete regime shifts. Specifically, $X_{t+1} - X_t = \kappa_0(\theta_0 - X_t) + \sigma_0\sqrt{X_t}u_{t+1}$ if the regime $s_{t+1} = 0$, and $X_{t+1} - X_t = \kappa_1(\theta_1 - X_t) + \sigma_1\sqrt{X_t}u_{t+1}$ if the regime $s_{t+1} = 1$. Note that the innovation in process (2), u_{t+1}, is conditionally normal given X_t and s_{t+1}. For analytical tractability we assume that the process for regime shifts s_{t+1} is independent of $X_{t+1-\tau}$, $\tau = 0, \ldots, \infty$. This is similar to the assumption made in Hamilton's regime-switching models. We also assume that the agents in the economy observe the regimes, although the econometrician may possibly not observe the regimes.

The pricing kernel for this economy is similar to that in standard models, except for incorporating regime shifts,

$$M_{t+1} = \exp\left\{-r_{f,t} - \left(\frac{\lambda_{s_{t+1}}}{\sigma_{s_{t+1}}}\right)^2 \frac{X_t}{2} - \frac{\lambda_{s_{t+1}}}{\sigma_{s_{t+1}}}\sqrt{X_t}u_{t+1}\right\}. \qquad (3)$$

The foregoing specification of the pricing kernel captures the intuition that these aggregate processes are latent and subject to regime shifts (as in Hamilton 1989). Note that the λ parameter that affects the risk premia on bonds is also subject to regime shifts and hence depends on s_{t+1}. Bansal and Zhou (2002) presented a general equilibrium model that leads to the pricing kernel in (3).

With regime shifts, we conjecture that the bond price with n periods to maturity at date t depends on the regime $s_t = i$, $i = 0, 1$, and X_t

$$P_i(t, n) = \exp\{-A_i(n) - B_i(n)X_t\}.$$

The one-period-ahead bond price, analogously, depends on s_{t+1} and X_{t+1},

$$P_{s_{t+1}}(t+1, n-1) = \exp\{-A_{s_{t+1}}(n-1) - B_{s_{t+1}}(n-1)X_{t+1}\}.$$

In addition, we impose the boundary condition $A_i(0) = B_i(0) = 0$ and the normalization $A_i(1) = 0$, $B_i(1) = 1$, for $i = 0, 1$; that is, $r_{f,t} = X_t$. The key asset pricing condition is

$$E_t\left[\mu_{n,s_{t+1},t} + \frac{\sigma^2_{n,s_{t+1},t}}{2} - r_{f,t}\middle| X_t, s_t\right] = -X_t E_t\left[B_{s_{t+1}}(n-1)\lambda_{s_{t+1}}\middle| s_t\right]. \qquad (4)$$

The conditional mean and volatility of the bond return in regime s_{t+1} are $\mu_{n,s_{t+1},t}$ and $\sigma^2_{n,s_{t+1},t}$. Equation (4) captures the idea that all risk premia and bond prices at date t depend only on s_t and X_t. To gain further intuition regarding this risk premium

Given information regarding s_t, X_t, and the regime transition probabilities, agents integrate out the future regime, s_{t+1}, which leads to the risk premium result stated in (4). In the absence of regime shifts, the risk premium in (4) would simply be $-X_tB(n-1)\lambda$. Hence incorporating regime shifts makes the "beta" of the asset (i.e., the coefficient on X_t) be time varying and dependent on the current regime. This fashion of making the asset "beta" time varying is potentially important for capturing the behavior of risk premia on bonds. In this model the market price of risk (i.e., the risk premium for an asset with a unit exposure to u_{t+1}) is $E_t[\lambda_{s_{t+1}}/\sigma_{s_{t+1}}|s_t]\sqrt{X_t}$, which is clearly regime dependent.

Given (4), the solution for the bond prices can be derived by solving for the unknown coefficients A and B. In particular,

$$\begin{bmatrix} B_0(n) \\ B_1(n) \end{bmatrix} = \begin{bmatrix} \pi_{00} & \pi_{01} \\ \pi_{10} & \pi_{11} \end{bmatrix} \times \begin{bmatrix} (1-\kappa_0-\lambda_0)B_0(n-1) - \frac{1}{2}\sigma_0^2 B_0^2(n-1) + 1 \\ (1-\kappa_1-\lambda_1)B_1(n-1) - \frac{1}{2}\sigma_1^2 B_1^2(n-1) + 1 \end{bmatrix} \tag{5}$$

and

$$\begin{bmatrix} A_0(n) \\ A_1(n) \end{bmatrix} = \begin{bmatrix} \pi_{00} & \pi_{01} \\ \pi_{10} & \pi_{11} \end{bmatrix} \begin{bmatrix} A_0(n-1) + \kappa_0\theta_0 B_0(n-1) \\ A_1(n-1) + \kappa_1\theta_1 B_1(n-1) \end{bmatrix}, \tag{6}$$

with initial conditions $A_0(0) = A_1(0) = B_0(0) = B_1(0) =$ Note that bond price coefficients are mutually dependent both regimes, current bond prices reflect the agent's expectations regarding regime shifts in the future. Finally, the bo yield of a K factor regime-shifting model can be derived i analogous manner,

$$Y_s(t,n) = -\frac{\ln P_s(t,n)}{n} = \frac{A_s(n)}{n} + \sum_{k=1}^{K} \frac{B_{ks}(n)X_{kt}}{n} \tag{7}$$

3. EMPIRICAL ESTIMATION AND MODEL EVALUATION

3.1 Estimation Methodology

To utilize a consistent approach for evaluation and estimation across the different models, we rely on the simulation-based EMM estimator developed by Bansal et al. (1995) and Gallant and Tauchen (1996). The EMM estimator comprises three steps. The first, the projection step, entails estimating a reduced-form model (the auxiliary model) that provides a good statistical description of the data. Multivariate bond yields are difficult data to model, because they exhibit extreme persistence in location and scale, time-varying correlations, and non-Gaussian innovations. Because we do not have good a priori information on the specifications of a model that captures all of these features, we utilize a seminonparametric (SNP) series expansion. The SNP expansion has a vector autoregressive–autoregressive conditional heteroscedasticity (VAR–ARCH) Gaussian density as its leading term, and the departures from the leading term are captured by a Hermite polynomial expansion. We elected to use a simpler, ARCH-like leading term instead of a generalized ARCH (GARCH)-type leading term because of the similar problems with multivariate GARCH-type models of bond yields noted by Ahn et al. (2002).

In the second step, the estimation step, the score function from the log-likelihood estimation of the auxiliary model is used to generate moments for a generalized method of moments (GMM)-type criterion function. The score function provides a set of moment conditions that are true by construction and are to be confronted by all term structure models under consideration. In the computations, the score function is averaged over the simulation output from a given term structure model and the criterion function is minimized with respect to the parameters of the term structure model under consideration. By using

Table 1. Summary Statistics

	1-month	3-month	6-month	1-year	2-year	3-year	4-year	5-year
Mean	5.9424	6.3765	6.5971	6.8106	7.0156	7.1711	7.2909	7.3545
Standard deviation	2.4499	2.5767	2.6038	2.5239	2.4559	2.3814	2.3491	2.3240
Skewness	1.4278	1.3717	1.3041	1.1737	1.1288	1.1283	1.1003	1.0565
Kurtosis	5.4659	5.1336	4.8778	4.4157	4.1226	4.0313	3.9196	3.7344

NOTE: There are 451 monthly observations of the yields with 8 maturities. The data are obtained from CRSP Treasury Bill and Bond files, ranging from June 1964 to December 2001.

verge to virtually any smooth distributions, including mixture distributions (as is the case with a model of regime shifts).

The third step is reprojection, or postestimation analysis of model simulations. Because EMM is a simulation-based estimator, long simulated realizations from each estimated model are available for analysis. These simulations can be used to compute statistics of interest that can be compared to analo-

sions, which, as stated earlier, provides potential economic motivation for incorporating regime shifts. The summary statistics of these monthly yields are displayed in Table 1. On average, the yield curve is upward sloping. The standard deviation, positive skewness, and kurtosis are systematically higher for short maturities than for long ones. To incorporate important time series and cross-sectional aspects of term structure data, we focus

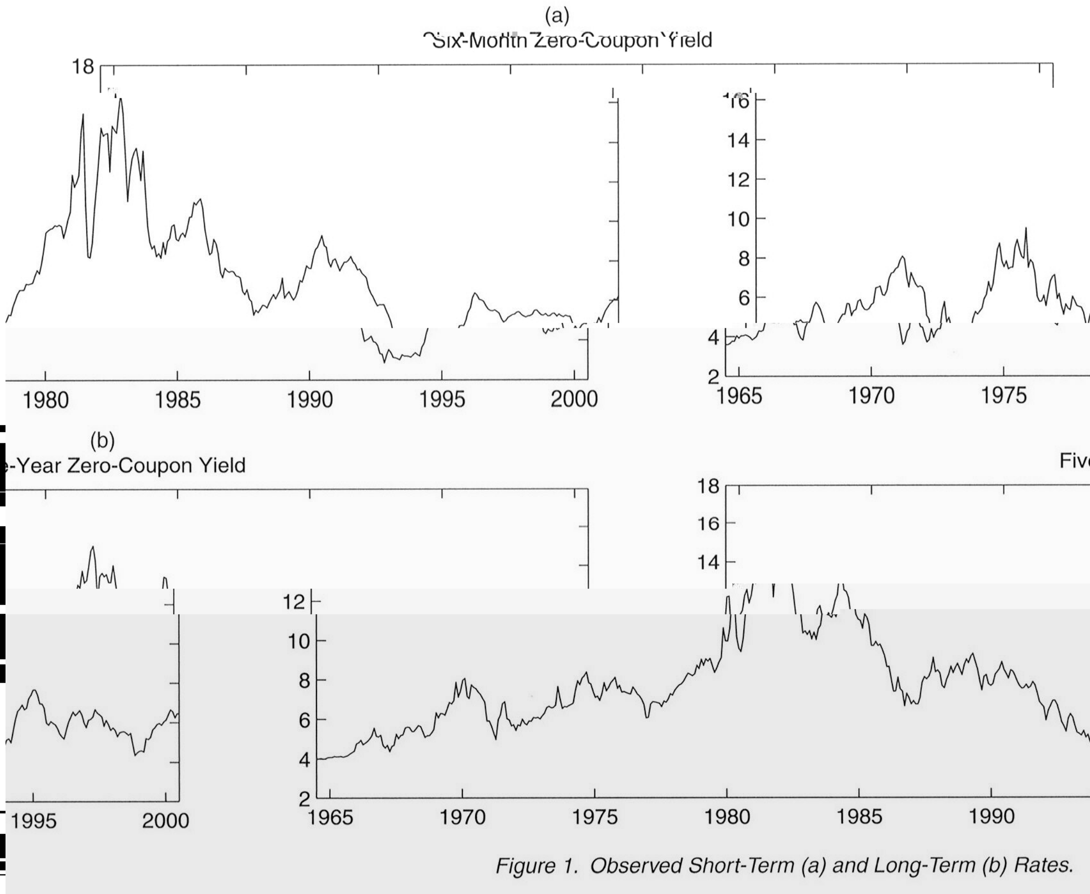

Figure 1. Observed Short-Term (a) and Long-Term (b) Rates.

in ARCH specification ($L_r = 5$). The preferred specification accommodates departures from conditional normality via a Hermite polynomial of degree 4 ($K_z = 4$). This "semiparametric ARCH" specification is similar to that proposed by Engle and González-Rivera (1991) and allows for skewness and kurtosis in the error distribution. The total number of parameters for the specification is $l_a = 28$; hence each model must confront a total of 28 data-determined moment conditions.

The conditional moments of the estimated SNP density for the observed interest rates are available analytically. It is fairly instructive to focus on some specific aspects of the estimated nonparametric SNP bivariate density. The top panel in Figure 6 (Sec. 3.6) gives the estimated conditional volatilities and cross-correlations of the 6-month and 5-year yields, which seem to be very persistent and fairly volatile. The 6-month yield has a wide range of conditional volatility that peaks around 1980, whereas the range for the 5-year yield volatility is narrow. The range for the conditional correlation is from about 40% to 80%, a wide range indeed. The most volatile period for bond yields, the early 1980s, is associated with a considerable drop in the conditional

directly on bond excess returns, and hence our estimation does not directly utilize information on the predictability of bond returns. We use the estimated model to evaluate via simulation, if the model can reproduce the predictability regressions discussed by Cochrane and Piazzesi (2002). These predictability regressions are challenging for two reasons. First, the size of the predictability is fairly high; the R^2's in these projections are quite large. Second, the nature of the predictability—the "tent shape" of the multiple regression coefficients—captures the unconditional covariation of future bond returns with current forward rates. A reasonable term structure model should account for both of these features of the predictability along with the important data aspects embodied in the bivariate SNP density for 6-month and 5-year yields.

3.3 Model Estimation Results

Table 2 gives the main EMM estimation results for four different models: one-factor regime-shifting (1-factor[RS]), two-

Table 2. Model Estimation by Efficient Method of Moments

	1-factor[RS]	2-factor[CIR]	2-factor[RS]	3-factor[AF]

pectation hypothesis puzzle and other data features of interest. The results reported here are for a simulation size of 50,000

[redacted]

Associated with this 3-factor[AF] specification are three market prices of risk parameters, which, as before, we label λ_k, $k = 1, 2, 3$. In all, there are 13 parameters to estimate. As reported in Table 2, the 3-factor[AF] specification is sharply rejected with $\chi^2(15) = 42.803$ and a p value of .0017. In a more general semiparametric setting, Ghysels and Ng (1998) rejected the affine restrictions on the conditional mean and variance of yields.

The 2-factor[RS] model can be viewed as a three-factor model with the regime-shifting factor being a multiplicative or nonlinear third factor. For a fair comparison of this model, we also estimated a three-factor affine term structure model, (3-factor[AF]), preferred by Dai and Singleton (2000), who found considerable empirical support for this specification using the post-1987 swap yield data. The discrete time counterpart to this affine specification is

$$X_{1t+1} - X_{1t} = \kappa_1(\theta_1 - X_{1t}) + \sigma_1\sqrt{X_{1t}}u_{1t+1},$$

$$X_{2t+1} - X_{2t} = \kappa_2(\theta_2 - X_{2t}) + \sigma_2 u_{2t+1} + \sigma_{23}\sqrt{X_{1t}}u_{3t+1}, \quad (8)$$

$$X_{3t+1} - X_{3t} = \kappa_3(X_{2t} - X_{3t}) + \sqrt{X_{1t}}u_{3t+1} + \sigma_{31}\sigma_1\sqrt{X_{1t}}u_{1t+1} + \sigma_{32}\sigma_2 u_{2t+1}.$$

Table 3. Diagnostic t-Ratios

Parameter	Description	1-factor[RS]	2-factor[CIR]	2-factor[RS]	3-factor[AF]
Hermite					
$A(1)$	00 00				
$A(2)$	01 00	.30	−1.038	−.752	.528
$A(3)$	10 00	2.13	.240	−.646	.898
$A(4)$	02 00	1.47	1.874	1.809	2.215
$A(5)$	11 00	−3.13	−2.258	1.251	−1.402
$A(6)$	20 00	2.36	−2.752	1.921	−1.538
$A(7)$	03 00	.08	−.072	−.152	1.431
$A(8)$	30 00	.40	−1.093	−.442	−.582
$A(9)$	04 00	1.05	2.018	1.634	2.384
$A(10)$	40 00	2.20	−1.230	1.423	−.389
Mean					
$\psi(1)$	$u(1)$	2.61	.263	−1.022	1.100
$\psi(2)$	$u(2)$	−.69	−.716	−.299	−.487
$\psi(3)$	$u(1)$, $y(1)$, lag 1	−1.75	.859	.963	.568
$\psi(4)$	$u(2)$, $y(1)$, lag 1	−.11	−.497	−.342	−.213
$\psi(5)$	$u(1)$, $y(2)$, lag 1	−2.31	.534	1.312	.017
$\psi(6)$	$u(2)$, $y(2)$, lag 1	[illegible]	−.054	[illegible]	[illegible]
ARCH					
$\tau(1)$	$R(1)$	1.85	−3.402	1.264	−2.140
$\tau(2)$	$R(2)$	−4.27	−2.924	.155	−2.692
$\tau(3)$	$R(3)$	3.98	3.579	1.369	2.962
$\tau(4)$	$R(1)$, $z(1)$, lag 5	2.56	−1.606	1.576	−.640
$\tau(9)$	$R(3)$, $z(2)$, lag 5	2.76	2.063	.104	1.641
$\tau(10)$	$R(1)$, $z(1)$, lag 4	2.57	−1.307	1.858	−.467
$\tau(15)$	$R(3)$, $z(2)$, lag 4	2.80	1.916	.933	1.891
$\tau(16)$	$R(1)$, $z(1)$, lag 3	1.68	−2.097	1.008	−1.621
$\tau(21)$	$R(3)$, $z(2)$, lag 3	4.41	3.474	1.963	3.198
$\tau(22)$	$R(1)$, $z(1)$, lag 2	2.99	−.212	1.644	−.003

[redacted]

The 1-factor[RS] m
2-factor[CIR] mode
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specifications is the
ing 1%. The estima
just under .95. All o
rather accurately. T
2-factor[RS] specifi
other authors (see G
The 2-factor[RS]
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or nonlinear third fa
we also estimated a
(3-factor[AF]), pref
found considerable

to estimate. As re- cation is sharply re- of .0017. In a more d Ng (1998) rejected can and variance of

the 28 moment conditions. These 28 scores tly specified model, under consideration sideration, then at a the t-ratio should be are not adjusted for before asymptotically They thus must be by the overall chi-one- or two-factor models, but it still has 4 out of 13 ARCH scores and 2 out of 9 Hermite scores that are not well matched. Overall, our preferred 2-factor[RS] specification seems to have

1991). Bansal and Zhou (2002) provided evidence that the two-factor regime-shifting model is the only one that can replicate this type of expectations hypothesis violation at the shorter ma-

Following the same conventions of Cochrane and Piazzesi (2002), we work with log bond prices (i.e., p_t^k is the log of the price at t of a k year bond), and geometric (log) yields and returns, so $y_t^1 = -p_t^1$ is the geometric yield on the 1-year bond. Cochrane and Piazzesi (2002) considered the regression of excess returns of bonds on the yields and the forward rates,

$$ex_{t+12}^k = \beta_{k0} + \beta_{k1} y_t^1 + \sum_{i=2}^{5} \beta_{ki} f_t^i + \epsilon_{t+12}^k, \qquad k = 2, \ldots, 5, \quad (9$$

where $ex_{t+12}^k = p_{t+12}^{k-1} - p_t^k - y_t^1$ is the excess return on the k yea[r] bond and $f_t^k = p_t^{k-1} - p_t^k$ is the forward rate. Note that ex_{t+1}^k is effectively the return on holding a k year bond for 1 year i[n] excess of the 1-year yield. This excess return data is collecte[d] monthly, which leads to the usual overlap in return data.

Table 4. *Predictability of Bond Excess Returns Using Multiple Forward Rates*

R^2	4-year	1-year, 3-year	1-year, 3-year, 5-year	1-year, 3- to 5-year	1- to 5-year
R^2's in the data					
2-year bond	.1744	.2619	.3088	.3187	.3280
3-year bond	.1322	.2538	.3326	.3357	.3373
4-year bond	.1368	.2634	.3406	.3617	.3639
5-year bond	.1297	.2640	.3163	.3308	.3336
Coefficient	*Intercept*	*1-year*	*3-year*	*5-year*	R^2
[redacted]					
5-year bond	28.6284	2.6990	−8.1863	6.2503	.2579

NOTE: The dependent variable in all of the regressions is the 1-year return from holding a bond with n years to maturity less the yield on a bond with one year to maturity. This annual excess return is tracked monthly. All R^2's are adjusted for degrees of freedom. The sample size in the data is 451 observations. In the top panel the predictability regression is run using 1-, 2-, 3-, 4-, and 5-year forward rates as regressors. Because the R^2 using 1-, 3-, 5-year forward rates is almost the same as using additional forward rates (see 1-, 3–5, and 1–5 years), we focus on the 1-, 3-, and 5-year projections. Newey–West robust standard errors are reported in parentheses in the "Regression Coefficients and R^2 in the Data" section for this projection. The results reported for the 1-factor[RS], 2-factor[CIR], 2-factor[RS], and 2-factor[AF] models are based on simulating 50,000 observations from the estimated term structure model and running the same regression as reported in the "Regression Coefficients and R^2 in the Data" section.

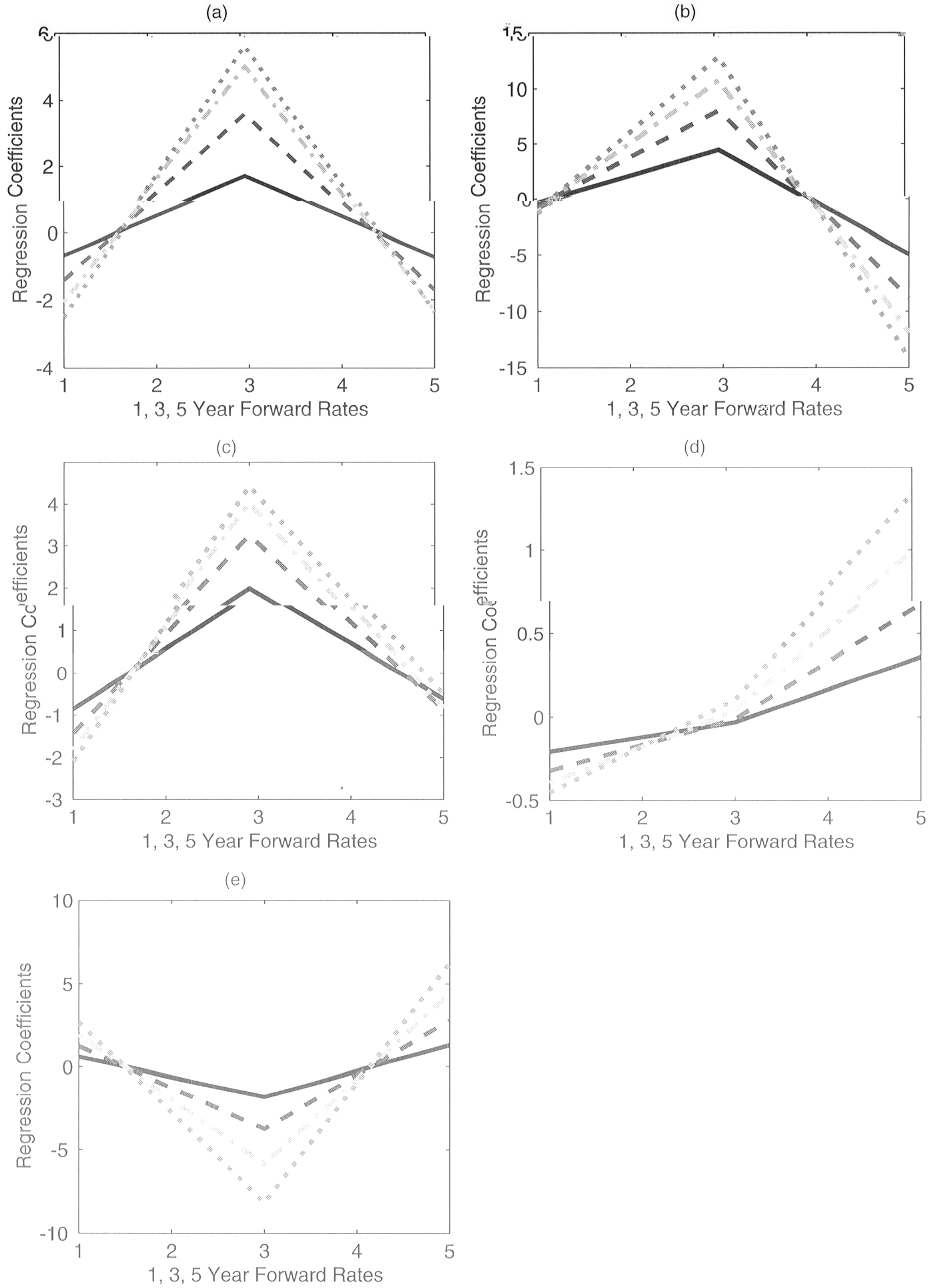

Figure 2. Predictability Regression Coefficients. (a) Observed data; (b) 1-factor[RS] model; (c) 2-factor[RS] model; (d) 2-factor[CIR] model; (e) 3-factor[AF] model (—— 2-year bond, 1-year excess return; – – 3-year bond, 1-year excess return; ···· 4-year bond, 1-year excess return; ···· 5-year bond, 1-year excess return).

timated parameters of the four models, we simulate 50,000 monthly data and run the same regressions of excess bond returns on forward rates. As seen in the lower panel of Table 4, the 2-factor[RS] model not only achieves the highest predicting R^2 (20–36%), but also clearly closely mimics the tent-shaped regression coefficients. On the other hand, the 2-factor[CIR] model produces a skewed and inverted tent shape, and the 3-factor[AF] model produces a inverted tent shape. Both models achieve R^2's around 10–20%. Interestingly, even the 1-factor[RS] model can replicate the tent shape to some degree, even though its R^2 is only about 1%. These patterns are quite apparent in Figure 2. These results suggest that the prediction capability of forward rates for excess returns may be explained by two or three linear factors, whereas the tent pattern of regression coefficients appears to be due to the regime-shifting nature of the yield curve.

The analysis of Duffee (2002) and Dai and Singleton (2002) suggest that allowing more flexible specification of the risk premium parameters for the conditional Gaussian factor model can dramatically improve its ability to match the predictability of excess returns. To explore this argument, we have also estimated the "preferred essentially affine $A_0(3)$ model" discussed by Duffee (2002) with three Gaussian factors and eight market-price-of-risk parameters (we call it the 3-factor[EA] model). The chi-squared test of overall specification is 29.278 with 9 degrees of freedom and a p value of .0006; hence the model is not supported by the data. The estimation result suggests that the 3-factor[EA] model overshoots the excess returns predictability, the R^2 range from 26% to 65% vis-a-vis 30% observed in the data. More importantly, it cannot reproduce the tent shape of the predictability regression coefficients. Further, its performance for cross-sectional pricing error is somewhat worse than that of the three-factor affine model. Our diagnostics for this model specification reveal that the implied conditional volatility and conditional correlations of yields do not match those in the data. Given this result, for brevity we do not present very detailed evidence on this specification.

3.5 Regime Indicator, Risk Premium, and the Business Cycle

We now explore the cross-sectional implications of the term structure models over the maturities that are not used in the model estimation. We also look at the association between the bond market implied regimes and the real business cycle. For the 2-factor[CIR] and 3-factor[AF] models, a standard method is used to calculate the pricing errors. Because the yield curve solution is linear in the factors, we first invert from two or three basis yields to get the latent factors and then use the linear pricing solution to calculate the nonbasis yields. For the 1-factor[RS] and 2-factor[RS] models, the presumption that agents in the economy know the current regime implies a strategy to recover the regimes. Specifically, dates are classified into regimes according to which of the two yield curves produces the smallest pricing error. Under the null of correct specifica-

Table 5. Average Absolute Pricing Error (basis points)

	1-factor[RS]	2-factor[CIR]	2-factor[RS]	3-factor[AF]
Mean	45	44	27	31
Median	34	40	19	23
Standard	33	24	22	28
Minimum	5	5	3	1
Maximum	223	156	154	188

[redacted]

Table 5 reports the time-series average of pricing errors $1/T\sum_{t=1}^{T}\mathrm{PE}_s(t)$ or other statistics from the cross-sectional average $\mathrm{PE}_s(t) = 1/N\sum_{n=1}^{N}|\hat{Y}_s(t,n) - Y_s(t,n)|$, where $\hat{Y}_s(t,n)$ is the calculated yield and $Y_s(t,n)$ is the observed yield for maturity n at time t (where the current state s is inferred from minimizing the pricing errors of the two yield curves, as mentioned earlier). It is clear from the sample statistics that the 2-factor[RS] model has the smallest average pricing error and also the smallest standard deviation in the pricing error. The maximal pricing error associated with the 2-factor[RS] specification is also the smallest. Further, on average the pricing error is only about 27 basis points for the annualized percentage yields. The 3-factor[AF] specifications have average pricing errors of 31 basis points, which in an absolute sense is also quite small. The 1-Factor[RS] and 2-factor[CIR] models achieve similar pricing results as 44 to 45 basis points.

It has been well recognized that the slope of the yield curve (i.e., spread) has the ability to predict future real GDP growth; in particular, negative spreads tend to predict a recession (see, e.g., Harvey 1988; Estrella and Hardouvelis 1991). Figure 3 recreates this linkage between the monthly yield spread, our regime indicator for regime 0 (our low regime), and the National Bureau of Economic Research (NBER) business cycles recession indicator. Most of the time, it seems that the economy is in regime 1. The total number of regime switches recovered from the sample period is 44. The regime relates to the NBER business cycles. Our low regime (regime 0) obtains during or before recessions in the economy. In the data, the correlation between NBER business cycle indicator and the yield spread (5-year yield minus 6-month yield) is 15%. In general, the yield curve becomes inverted (or flat) several months before the economic growth becomes negative (or depressed). Our regime indicator is mostly 0, as Figure 3 shows, when the yield curve becomes inverted (or flat). The correlation between the model-based regime indicator and the yield spread (5-year yield minus 6-month yield) is 24%; that is, our high regime (regime 1) coincides with a high yield spread and our low regime (regime 0) largely coincides with a low yield spread. Therefore, as reported by Bansal and Zhou (2002), the regime indicator has the power to predict recessions. The correlation between the NBER business cycle (NBER recession as regime 0 and NBER boom as regime 1) and our regime indicator is .1117. In the context of modeling the short interest rate, Ang and Bekaert (2002) also documented the links between regime shifts and business cycles.

[redacted]

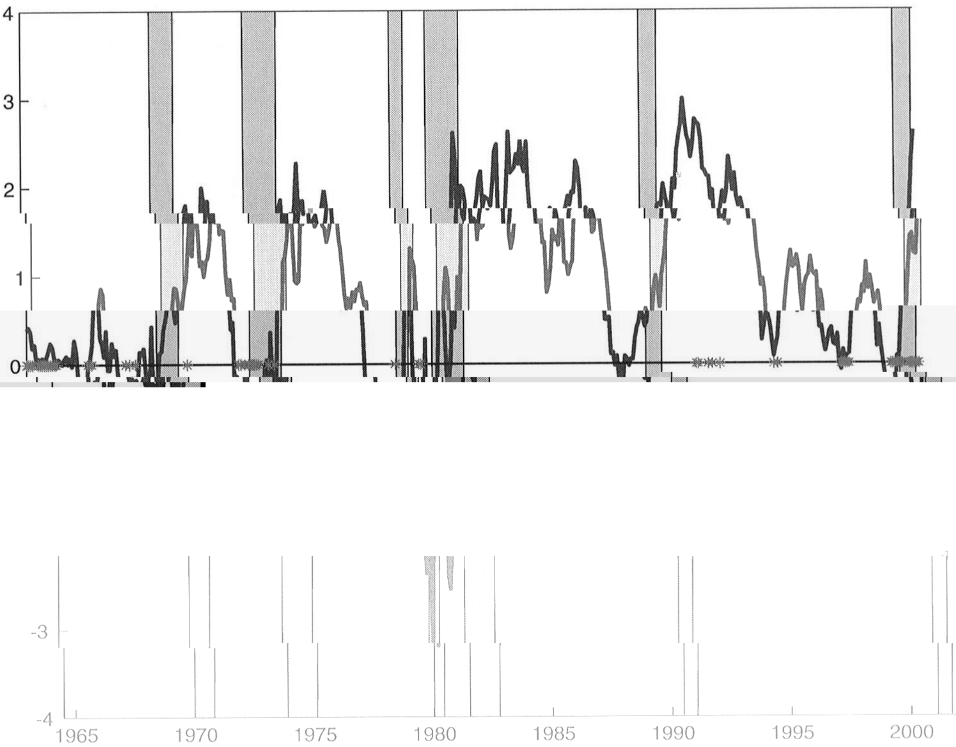

Figure 3. Yield Spread, Regime Indicator, and Business Cycle. The thick line is the 5-year yield minus the 6-month yield (yield spread), the shaded area is the NBER recession period, and the star is the indicator of our low regime (regime 0) from our preferred 2-factor[RS] model. The high regime (regime 1) corresponds to all dates without the star.

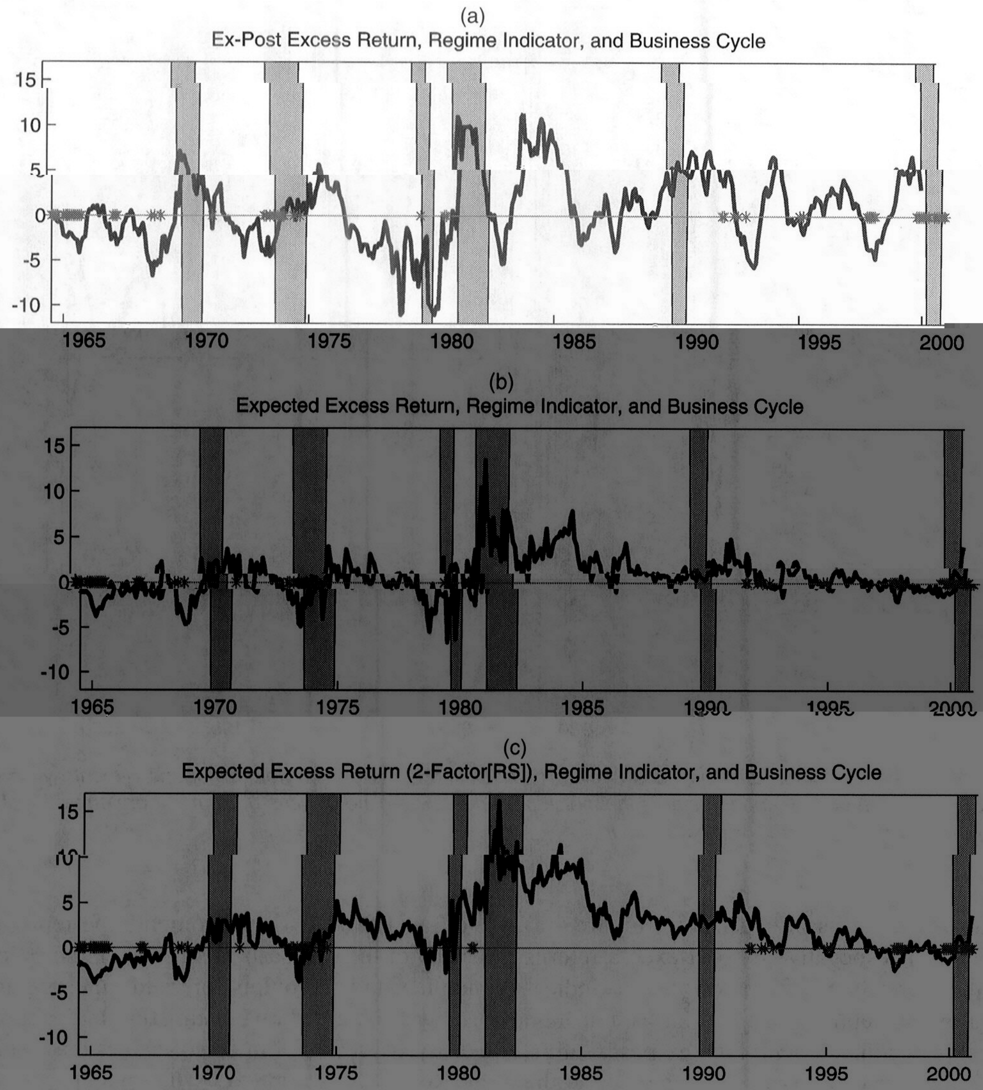

Figure 4. Excess Return, Regime Indicator, and the Business Cycle. The shaded area is the NBER recession period, and the star is the indicato of the low regime (regime 0) from our preferred regime-shifting term structure model. The thick line represent the annual ex-post excess return (a the expected excess return based on projecting future ex-post excess returns on three forward rates (b), and the reprojected expected exce return from our 2-factor[RS] model (c). All ex-post and expected excess returns are averages (across bonds) using the 2- to 5-year bonds.

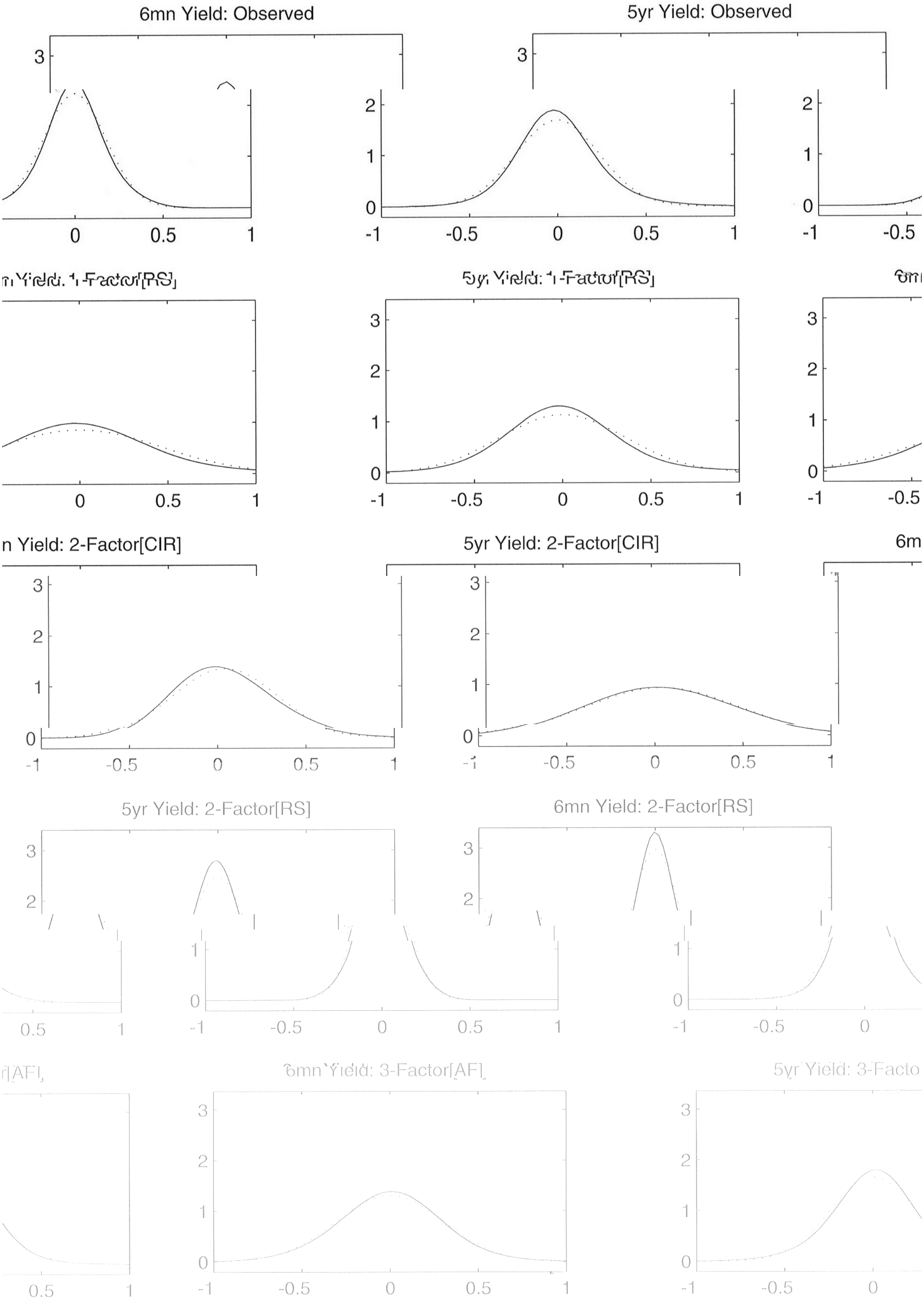

Figure 5. Reprojected Densities.

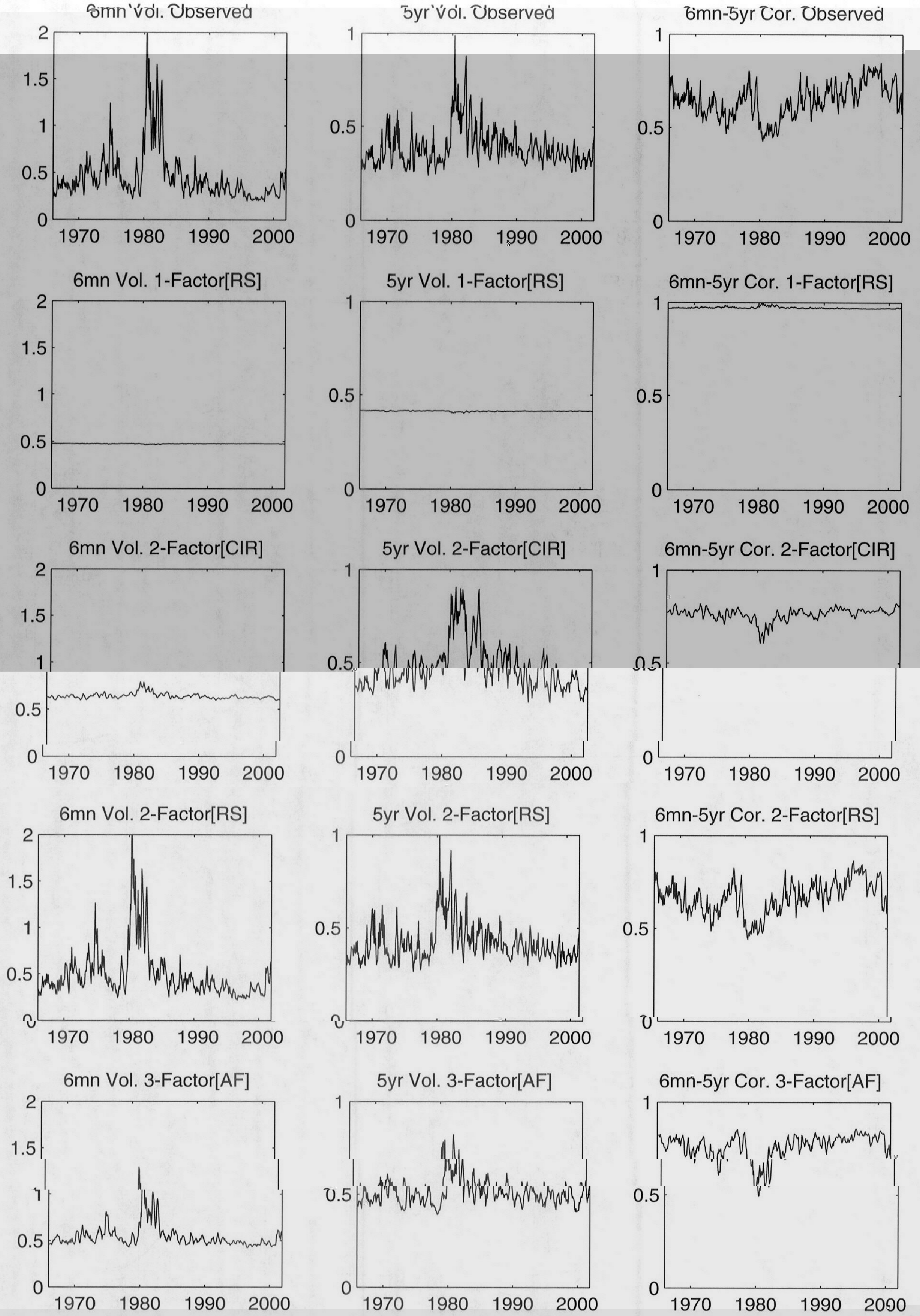

Figure 6. Reprojected Volatilities and Correlations.

4. CONCLUDING REMARKS

Business cycle movements between economic expansions and recessions affect macroeconomic variables, financial markets, and, in particular, the term structure of interest rates. In this article we have incorporated the well-documented feature of regime shifts as given by Hamilton (1988) into the standard

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Cai, J. (1994), "A Markov Model of Switching-Regime ARCH," *Journal of*