Urbanization in a Two-Sector Model with Sticky Wage and Firm Speci…c Capital in Urban Sector

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Abstract

A two-sector model is presented here to study the urbanization process in less developed dual economy. In di¤erence from existing literature, recent new Keynesian theory of sticky wage and …rm speci…c capital are applied to the urban sector of the model. This allows us to investigate the development strategy, such as comparaive advantage, among others in the urbanization process. Policies and institutional implications can be derived from analyzing the model.

Keywords: two-second model, urbanization, comparative advantage, sticky wage, …rm speci…c capital

JEL: O1, E2, O4

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1 Introduction

Less developed economies are often characterized as being agriculture (or rural) economy. Usually, the population in rural area is much larger than in urban area despite its low productivity relative to the productivity in urban area.[1] This indicates that the allocation of labor force are not e¢ cient. Thus, the development process of a less developed economy (once it starts to develope or to be in convergency path) often appears to be a process of urbanization.

Large majority of existing literature study the urbanization process in a two-sector (ubran sector and rural sector) model.[2] Two di¤erent production sector is important in explaining urbanization. For instance, the real wage in urban sector is important in determining the choice of production mode via investment. Most existing literature with regard to urbanization set the wage in a competitive way with the possible minimun wage restriction to address unemployment in urban sector of a less developed economy. In this paper, we adopt new Keynesian principle of sticky wage to formulate the wage determination in urban sector.

With the introduction of firm specific capital and sticky wage into the urban sector of our two-sector model, we find some robust results which are consistent with empirical evidence and some well-established theory in development economics.

The paper is organized as follows. In section 2, we will establish a model of firm specific capital, from which we derive an independent investment function. Section 3 presents our two-sector model. The urbanization process will be analyzed in section 4 against the model we establish in section 3. Section 5 reviews some empirical evidence and well-etablished theory in development economics that supports the results we derive from section 4. The mathematical appendix provides the proof of the proporsitions in text.

2 The decision on investment

The economy that we are considering is divided into two sector: urban sector and rural sector. The urban sector produces output with the standard Cobb-Dauglass production function. Thus, investment (or capital accumulation) is one of the key decisions made by an urban firm at the begining of each period t. In this section we shall discusses the investment decision by a representative firm i in urban sector of the economy.

2.1 Technology

Investment is to establish a new facility (or a firm's specifical capital). A facility K is measured by at least two elements: one is its capacity measured by the output Y produced at a normal service intensity of K; the other is

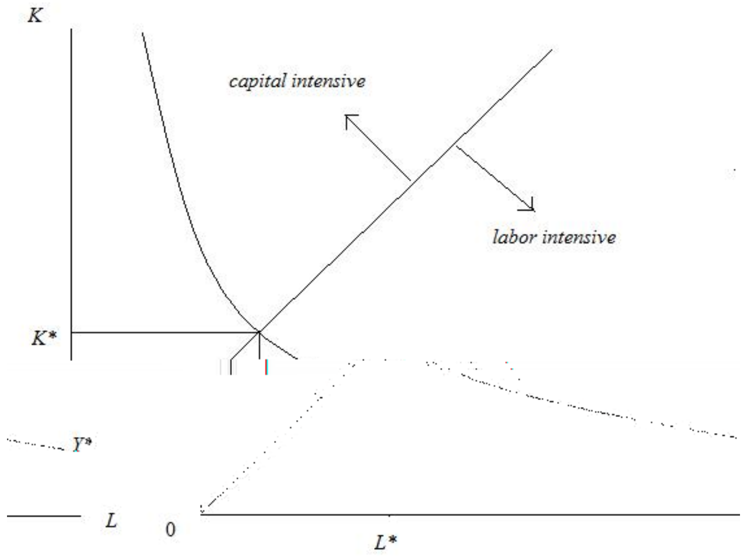

Figure 1: The Choice of Technology under Cobb-Dauglass Production Function

the technology measured by the input-output relation as requested by the facility. At the given production scale (or capacity) Y, we assume that a firm has a variety of choices on technologies so that its production function takes the form of Cobb-Dauglass:

$$Y = K^{1-\alpha}(XL)^{\alpha} \tag{1}$$

where L is the labor input and X is a coefficient measuring the level of advancement in technology so that XL can be regarded as labor in efficiency. Figure 1 provides a description of the technology at a sepecific level of X as implied by the Cobb-Dauglass production function.

In Figure 1, we assume that the firm for some reasons have eventually established a a facility K^*. That facility has the production scale Y^* and a technology that reflects a labor-output ratio

$$n^* = \frac{L^*}{Y^*} \tag{2}$$

Given K^*, Y^* and X

2.3 The ...nancial cost

In addition to the production cost as expressed by (6), the ...rm also generates the ...nancial cost from investment. We assume that the ...rm may borrow the money from ...nancial intermediaries to ...nance its investment I . Thus in each period, it also has to pay the interests and the loan principle that has been due. Let FC denote the ...nancial cost in period t resulting from the investment of the current and the past periods. It can easily be found that

$$FC = r I + r_{-1} I_{-1} + r_{-2} I_{-2} + \cdots + r_{-} I_{-} + I_{-}$$

where r is the interest rate in t, and τ is the length of loan periods. The following proposition regards this ...nancial cost from investment.

Proposition 1 Let $\beta \in (0, 1)$ denote the discount factor. Assume that the length of loan period is large enough so that $\beta \approx 0$ and $1 + \beta + \beta^2 + \cdots + \beta \approx$

A $\frac{1}{1-}$. Then, the following equation must hold:

$$\sum_{=0}^{\infty} \beta \, FC_{+} \approx F + \frac{1}{1-\beta} \sum_{=0}^{\infty} \beta \, r_{+} I_{+} \tag{8}$$

where F is the ...nancial cost resulting from the past investment I_{-1}, I_{-2}, ..., I_{-}.

We shall remark that F in the proposition can be regarded as a sunk cost when the ...rm makes its investment decision in period t. Therefore, it does not impact the ...rm's decision on I , I_{+1}, ..., as long as ...rm i is ...nanically solvable.

2.4 The decision problem

Given the production and the ...nancial cost, we are now able to formalize the problem of investment decision. We assume that the ...rm be given a sequence of expected demand $E\{Y_{+}\}_{=0}^{\infty}$ measured in real term. The problem can thus be expressed as choosing a sequence of investments $\{I_{\ +}\}^{\infty}_{=0}$ to

$$\max E \sum_{=0}^{\infty} \beta \ (Y_{\ +} - TC_{\ +} - FC_{\ +}) \tag{9}$$

subject to (6) - (8) and

$$K_{\ +} = (1-d)K_{\ +\ -1} + I_{\ +} \qquad d \in (0,1) \tag{10}$$

where d is the depriciation rate. We shall remark that although the …rm chooses a sequence of investment $\{I_{\ +}\}^{\infty}_{=0}$ only I is carried in period t. Proposition 1 regards the …rst-order condition with respect to the investment decision problem.

Proposition 2 The problem (9) subject to to (6) - (8) and (10) allows us to obtain

$$\frac{Y_{\ +}}{K_{\ +}} = \left[\frac{r_{+} - \beta(1-d)r_{\ +\ +1}}{w_{+\ -1}} b\right] \tag{11}$$

where $w_{+} = W_{+}/X_{+}$ and $b = \alpha x/[(1-\beta)(1-\alpha)]$.

Given Proposition 2, we are now able to derive the investment I . Write the …rst-order condition (11) for period t as

$$\frac{Y}{K} = f(r\ , r_{\ +1}, w_{\ -1}) \tag{12}$$

where

$$f(r\ , r_{\ +1}, w_{\ -1}) = \left[\frac{r\ - \beta(1-d)r_{\ +1}}{w_{\ -1}} b\right] \tag{13}$$

Above, $w \equiv W/X$ can be regarded as the real wage rate for labor in e¢ ciency. Since investment I is made to achieve the optimum output-capital ratio $f(\cdot)$ at the given production scale Y , the following condition should be satis…ed:

$$\frac{Y}{(1-d)K_{\ -1} + I} = f(r\ , r_{\ +1}, w_{\ -1})$$

From this, we derive

$$I \quad = \frac{Y}{f(r\ , r_{\ +1}, w_{\ -1})} - (1 - d)K_{\ \ -1} \tag{14}$$

Investment in period t thus depends on the demand Y , existing capacity $K_{\ \ -1}$, the current interest rate r , the future interest rate $r_{\ +1}$ and the real wage rate for e¢ ciency labor $w_{\ -1}$.

2.5 Economic explanation

From equation (14), it is apparent that Y has positive, and $K_{\ \ -1}$ has negative impact on investment I . Let us now discuss how investment is impacted by the other variables, that is, r , $r_{\ +1}$ and $w_{\ -1}$. For this we shall discuss the economic meaning of the optimum (or target) output-capital ratio $f(\cdot)$.

It should be noted that $f(\cdot)$ can also be regarded as a measure whether the technology choosed by the ...rm is capital-intensive or labor-intensive: a lower $f(\cdot)$ means higher facilitiy used for less output. Therefore it is capital-intensive. On the other hand, if $f(\cdot)$ is higher, it means less facility used for higher output and therefore it is labor-intensive. From (13), one ...rst ...nd that

$$\frac{\partial f}{\partial r} > 0, \quad \frac{\partial f}{\partial r_{\ +1}} < 0, \quad \frac{\partial f}{\partial w_{\ -1}} < 0$$

Since interest rate r can be regarded as the cost of capital stock, thus when interest rate is higher, the ...rm will choose more labor-intensive technology, indicating higher $f(\cdot)$ so that $\frac{\ }{_t} > 0$. Given existing capital stock $K_{\ \ -1}$ and the output Y to produce, this further indicates lower investment. Therefore the current interest rate r will have negative impact on investment I . Yet, due to $\frac{\ }{_{t+1}} < 0$, the future interest rate $r_{\ +1}$ will have positive impact on current investment I . This is because when the ...rm ...nds that the future interest rate is higher, it will increase investment now to avoid the possible higher investment cost in the future.

When wage rate $w_{\ -1}$ (in e¢ ciency) is higher, the ...rm will prefer more capital-intensive technology so that $f(\cdot)$ is lower. Given existing capital stock

$K_{\ -1}$ and the output Y to produce, this further indicates higher investment. Therefore, wage $w_{\ -1}$ has positive impact on investment other things equal.

3 The Model

Given the investment decision as discussed previously, we shall now formalize a simple two-sector macrodynamic model of a less developed economy. The economy is supposed to be a small open-economy. This indicates that output prices are determined by international market so that all the economic variables in our model is measured in real term by international price. In addition, we also do not have to consider the composition of domestic outputs produced by rural and urban sectors. At the given level of aggregate output produced by domestic economy, the demand for consumption (including the preference on aggriculture and manufacturing goods) and investment are all supposed to be satis…ed via international trade. Let us …rst discuss the wage setting in urban sector.

3.1 The wage setting in urban sector

The labor market is supposed to be seperated between urban and rural area. This indicates that the wage setting in these two areas are di¤erent. Let us …rst discuss the wage setting in urban sector.

In urban sector, the wage is set according to New Kaynesian way of sticky wage. As in Christiano, et al. (2005) and Erceg et al. (2000), we assume that a representative household h is a monopoly supplier of a di¤erentiated labor serive N . He (or she) thus sets the wage rate W at the begining of period t according to the demand curve for the labor service N :

$$W = \left(\frac{N}{\hat{N}}\right)^{-} \hat{W}, \quad \varepsilon \in (0,1) \tag{15}$$

where $\hat{N}$ is the expectation on N, the aggregate employment, measured as a proportion to the total labor force in urban area; $\hat{W}$ is the expectation on W, the aggregate real wage in urban sector. We remark that in the current literature it is often assumed that the household take W and N as given. Since W and N are themselves in the components of W and N via

$$W = \left(\int_0^1 (W\ )^{(\ -1)}\ dh\right)^{(\ -1)} \qquad N = \left(\int_0^1 (N\ )^{1-}\ dh\right)^{1\ (1-\ )}$$

we find that W and N will not be determined before the determination of W and N . This suggests that it is perhaps not unreasonable to use $\hat{W}$ and $\hat{N}$ instead of W and N in the household's perceived demand function (15).

We assume that the labor serive N be descrete only at 0 or 1. When N is zeros, it simply means that the household is unemployed. Otherwise it is employed at full time.

Next, we shall discuss the dynamics of aggreagte wage rate W. We assume that the dynamics of W follow the rule similiar to Calvo (1983) so that in each period, the household (who are employed in the last period) faces a constant probability $1-\theta$ of being able to reoptimize his (or her) real wage.[3] The ability to reoptimize is independent across households and time. If a housenold h cannot reoptimize his (or her) real wage at time t, he (or she) simply index the wage according to the rule of

$$W\ = \lambda\, W_{-1} \tag{16}$$

Christiano, et al. (2005) assumes that λ is equal to the lagged inflation rate. Since our paper is focused on the growth issue so that money and thus prices are not introduced, we shall assume that $\lambda\ = x$, where x is the gross growth rate of X , the technology. In consist

Given this discussion, the dynamics of wage rate W can be written as

$$W = \theta N_{-1} x W_{-1} + [(1-\theta)N_{-1} + 1 - N_{-1}] \left(N \right.$$

3.2 The output and capital stock in urban sector

Next, we shall discuss the output and capital stock in urban sector. To simplify our analysis, we shall first assume that $r = r_{+1} = r$ so that we do not have to consider the monetary policy to adjust the real interest rate. In this case, equation (13) can be written as

$$f(w_{-1}) = \left(\frac{q}{w_{-1}}\right)^{\partial} \qquad (20)$$

with $q = [1 - \beta(1 - d)]\, rb$. We notice that under the assumption of identical technology accross firms, the function f is independent from i, and therefore all the firms in period t should keep the same capital-output ratio.

In what follows, we shall make a somewhat strong assumption. We shall assume that all the firms in urban sector run at normal service intensity of their capital stocks. This indcates that we are not going to consider the business cycles issue in the paper. Denote $Y = \sum Y$ as the aggregate output produced by urban sector and $K = \sum K$ as the aggregate capital stock, we thus find from (12) that

$$Y = f(w_{-1})K \qquad (21)$$

By relying on (20), equation (21) allows us to derive

$$y = \left(\frac{w_{-2}}{w_{-1}}\right)^{\partial} k \qquad (22)$$

where $y \equiv \frac{\ _{I,t}}{\ _{I,t-1}}$ and $k \equiv \frac{\ _{t}}{\ _{t-1}}$ are respectively the gross growth rates of output Y and capital stock K. The accumulation of aggregate capital stock can be written as

$$K = (1 - d)K_{-1} + I$$

Expressing I in terms of sY, with s to be the saving ratio, while Y in terms of (21), we obtain from the above

$$K = (1-d)K_{-1} + sf(w_{-1})K$$

Dividng both sides by K_{-1} and re-organizing, we obtain

$$k = \frac{1-d}{1-sf(w_{-1})} \tag{23}$$

3.3 Employment in urban sector

Given the aggregate production Y , the aggregate demand for labor in urban sector L can be expressed as

$$L = n\,Y$$

where n is the labor-output ratio. From (2), (3) and (12), we …nd that

$$n = f(w_{-1})^{\frac{1-\alpha}{\alpha}} \frac{1}{X}$$

Let L denote the labor supply in urban sector. Thus, the employment rate N in urban sector can be written as

$$N = \frac{n\,Y}{L}$$

This equation allows us to obtain

$$\frac{N}{N_{-1}}^{\partial} = \left(\frac{w_{-2}}{w_{-1}}\right)^{1-} \frac{y}{xl} \tag{24}$$

where $l \equiv \frac{{}^{s}_{I,t}}{{}^{s}_{I,t-1}}$ is the gross growth rate of labor supply in urban sector, which can be regarded as a measure of migration.

3.4 The rural sector

In the rural area, all farmers are supposed to be self-employed. The production function in the rural sector takes the form

$$Y = \kappa X \left(LA\right), \quad \kappa, \gamma \in (0,1) \qquad I \tag{25}$$

where Y is the output produced by rural sector, LA is the labor in rural sector with A to be the proportion of total labor in rural sector and L to be the aggregate labor supply in the economy, which is assumed to be fixed in this model. Note that X is the economy-wide productivity that is influenced by the factors such as the state of scientific knowledge, market institutions, public infrastructure and government policy. The productivity in the rural area is thus linked to X through the parameter κ, which can be interpreted as "measuring the integration of agriculture to the aggregate economy"(Restuccia, et. al. 2007).

Equation (25) indicates that

$$y = x\left(\frac{A}{A_{-1}}\right) \qquad I \tag{26}$$

where y is the gross growth rate of agriculture product. Since farmers are self-employed, Y can also be regarded as the aggregate real income owned by the farmer. Dividing both sides of (25) by $X\, LA$, the everage income in efficiency can thus be written as

$$w = \kappa\left(LA\right)^{-1} \qquad I \tag{27}$$

This w is compared to w and determines the migration l, the gross growth rate of labor supply in urban sector. One may easily find that l can be written as

$$l = \frac{L - LA}{L - LA_{-1}} = \frac{1 - A}{1 - A_{-1}}$$

from which we obtain

$$A = 1 - l \quad (1 - A_{-1}) \tag{28}$$

Note that l reflects the speed of migration and therefore it is a function of wage differential:

$$l = 1 + \epsilon(w_{-1} - w_{-1}) \tag{29}$$

where ϵ can be regarded as a measure of barrier to migration (or reallociation cost). That barrier is determined by institution, policy, among others. A well known example of such barrier in China is the residential registration system (hu kou), which generates the variety of discriminations (in terms of schooling, social security, among others) against the families migrating from rural area.

The model has now been closed, which is composed of (19), (20), (22) - (24), (26) - (29).

3.5 The equilibrium

The equilbrium of the economy can be defined as the state at which no migration occurs. From (29), this requests that $w_{-1} = w_{-1}$ so that $l = 1$. The following proposition regards the equilibrium (or the steady state) of the two-sector economy.

Proposition 3 Let (k, y, y, N, w, w, A, l) denote the steady states of (k, y, y, N, w, w, A, l). Meanwhile let $f \equiv f(w)$. Suppose the total labor supply L k

y

The steady state of this economy is very much standard: not only the full employment is warranted in urban sector but also the economy is growing at the balanced growth path with the growth rate equal to the natural rate (that is, the rate of technical progress x).

4 Analysis on Urbanization

4.1 The structural parameters

To analyze the model, we shall first define the structure parameters in Table 1.

Table 1: Benchmark Parameters

d	β	r	x	α	γ	s	θ	ε	ϵ	κ
0.1	$\frac{1}{1+0\ 04}$	0.06	1.03	0.66	0.66	0.3	0.24	0.05	0.10	0.50

The parameters d, β, and r are assumed to be the annualized depriciation rate, discount factor and interest rate; x is the gross growth rate in productivity, which is consistent with the current gross growth rate of GDP in developed economies; α and γ are assumed to be the proportion of non-profit income; s is the saving ratio. All these are supposed to be the standard numbers. We assume that each quarter 70% employee are sticky at their previous wage.[5] This indicates that θ = 0.7^4. The value of ε simply borrows from Christiano, et. al (2005). The other two parameters ϵ and κ are heavily impacted by policies and therefore different values will be considered later in our analysis.

The steady states at these parameters are computed as in Table 2.

Table 2: The Steady States at Benchmark Parameters

N	k	y	y	l	f	w	w	A
1.00	1.03	1.03	1.03	1.00	0.421	1.606	1.606	0.032

[5] We shall remark that this number is varied substantially as indicated in Christiano, et. al (2005) from 0.42 to 0.80.

We will illustrate the dynamic behavior of our two-sector model with the benchmark parameters given in Table 1. The results are however robust to the perturbation of these parameters as long as they are within reasonable domains. Indeed, by repeated experiements in changing the structure paramters within reasonable domains, we do find that not only the convergence to the steady states is warranted, but also the dynamic behavior (that is, the basic mechanism of urbanization) as expressed below will remain the same.

4.2 Urbanization process, the benchmark case

Figure 2 provides a simulation to the urbanization process of our two-sector economy with the benchmark parameters given in Table 1.

The economy is assume to have the total labor supply normalized at 1. All variables are initially set at their steady states except A_0 and w_0. We set A_0 at 0.8, indicating 80% people live initially in rural area. Given such an initial condition, the initial real income per unit efficiency labor in rural sector w_0 is equal to $\kappa 0.8^{-1}$ = 0.5394. That real wage is much smaller than the wage in the urban sector, which we assume to be at its steady state (see Table 2). Thus, the urbanization starts with the wage difference between rural and urban sector.

The higher wage in urban sector induce farmers to migrate to cities so that the speed of migration is higher at the begining (see Panel E). This increases the labor supply and thus causes unemployment in cities (see Panel D). Large unemployment indicates that many people are entering the labor market for competing jobs while only small portion are sticky at their previous wage. Therefore the overall wage rate will be reduced fast (see the solid line in Panel C). The reduction in the real wage in urban sector will induce the firms in urban sector to choose the technology with higher output-capital ratio f or more labor-intensive technology (see Panel B). The higher output-capital ratio indicates that investment will be more productive in creating capacity. Therefore output can be increased fast in urban sector (see the solid line in

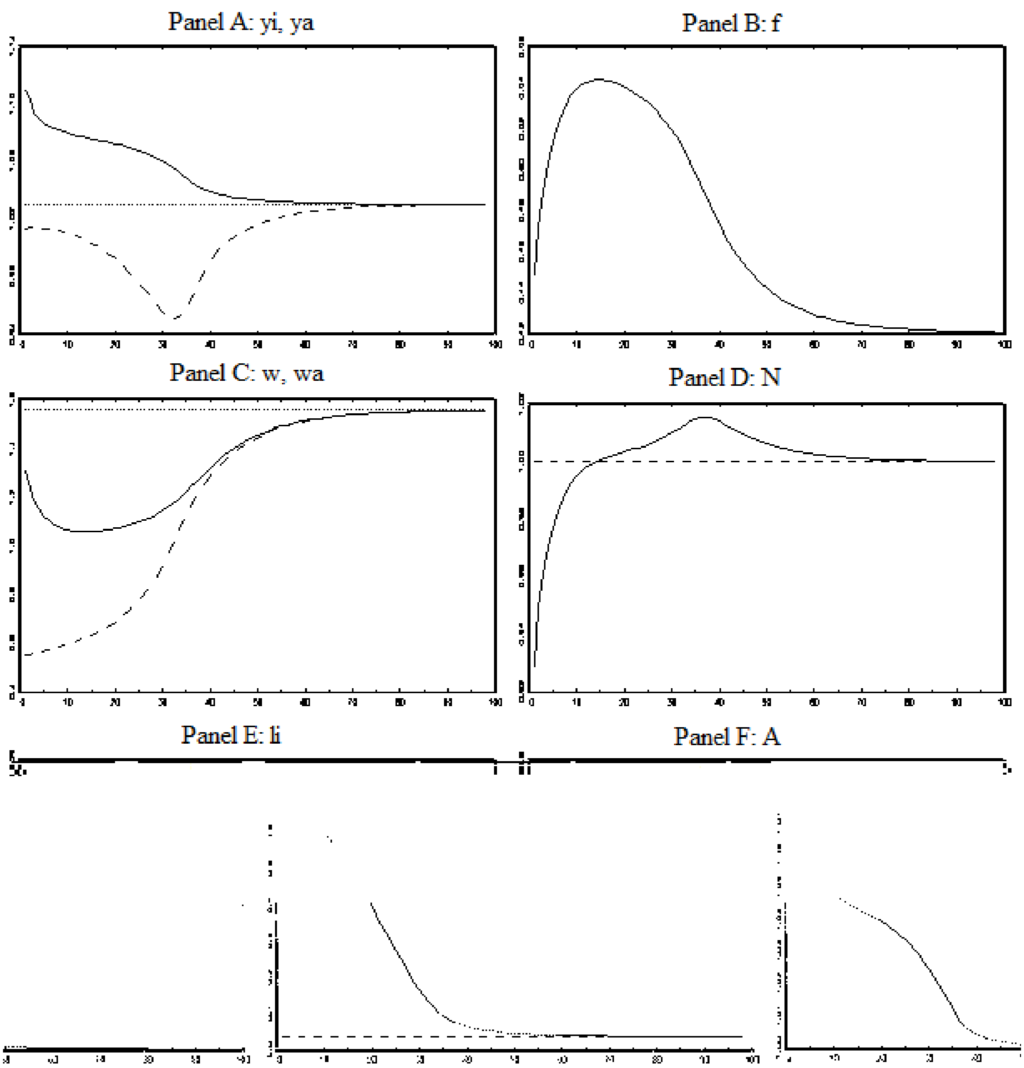

Figure 2: Urbanization Process with Benchmark Paramteters

Panel A)[6] though the gross growth rate of output in rural sector is reduced temporally due to less labor working in rural sector (see the dashed line in Panel A). On the other hand, the labor-intensive technology indicates that the job creation is more e¢ cient at the given output to produce. Therefore, employment rate in urban sector can be recovered (see Pane D).

With the continous migration, the labor supply in rural sector is gradually reduced (see Panel F). This increases the marginal product of labor and thus make the real income per unit e¢ ciency labor in rural sector increased (see the dashed Panel C) and thus the di¤erence of labor income between rural sector and urban sector is shrinked. Once the di¤erence is equal to zero, the migration will stop. Eventuall, all the variables are convergent to their steady states.

4.3 Distortion in the choice of technology

The urbanization process discussed previously can be understood as a benchmark case in which there is no distortation in the choice of technology. Yet in reality, the choice of technology can be distorted. The distortion in choosing technology may come from two sources: one from feasibility in technology and the other from government intervention.

As shown in Figure 1, the isoquant curves Y^*, indeed, any other isoquant curve, is not only smooth, but also extend asymontotically towards the two axis K and L, that is, $[0, +\infty)$ and $(+\infty, 0]$. In other words there exists a mode of production that makes f be in…nitively large regardless the scale of production. It is this limitless in f that makes the pro…t maximization …rm always able to …nd a mode of production that is most suitable at the margin to the existing wage w. Thus if we impose a restriction on the output-capital ratio f, which does not seem to be much unreasonable, the choice of technology via f will be distorted.

The second distortion may come from government: a govenment may set a restriction on the choice of technology that violates the market principle (and thus also the principle of comparative advantage). This has occured in

[6]given su¢ cient demand, which is assumed to be manageable by government policy.

many developing countries, including China.[7]

Let us now set a restriction f^* on the output-capital ratio f so that $f < f^*$. This restriction may result from the feasiblity of technology or from government intervention. In Figure 3, we add a simulation, re‡ected by the dashed line, of the urbanization process when f^* = 0.48 while the solid line still re‡ects the benchmark case as in Figure 2.

We …rst notice that setting restriction f^* does not impact the steady state of the economy so that the two lines in Figure 3 will eventually converge. Yet when f is restricted, investment will be less productive in creating capacity. Thus, other things equal, the growth in output will be slower (see Panel A: notice that we no longer presents here with the growth rate of output produced by rural sectors). The restricted f also indicates that the products in urban sector are less-labor intensive. Therefore, job creation in urban sector will be slowly and unemployment will be more serious at the earilier stage of development (see the dashed line in Panel D). The more serious unemployment will lead lower wage in urban (see the dashed line in Panel C, note that we no longer present here the wage in rural sector) and thus its di¤erence from rural sector is small. The speed of migration is reduced, that is, l is lower at the begining (see the dashed line in Panel E). All in all, the urbanization will proceed slowly since A declines slowly (see the dashed line in Panel F).

We thus …nd that the distortion to the choice of technology is not beni…t to the economy in terms of fast urbanization.

4.4 The barrier to migration

In the previous benchmark model, we have assumed that ϵ = 0.1. As we have mentioned before, this parameter may heavily be impacted by the policies with regard to migration. Therefore, it might be interesting to look at the urbanization process when we set ϵ di¤erently. In Figure 4 we add a simulation, re‡ected by the dashed line, of the urbanization process when ϵ = 0.05 (that is, the government sets more barrier to migration) while the solid line

[7] More discussion will be made in the last section of this paper.

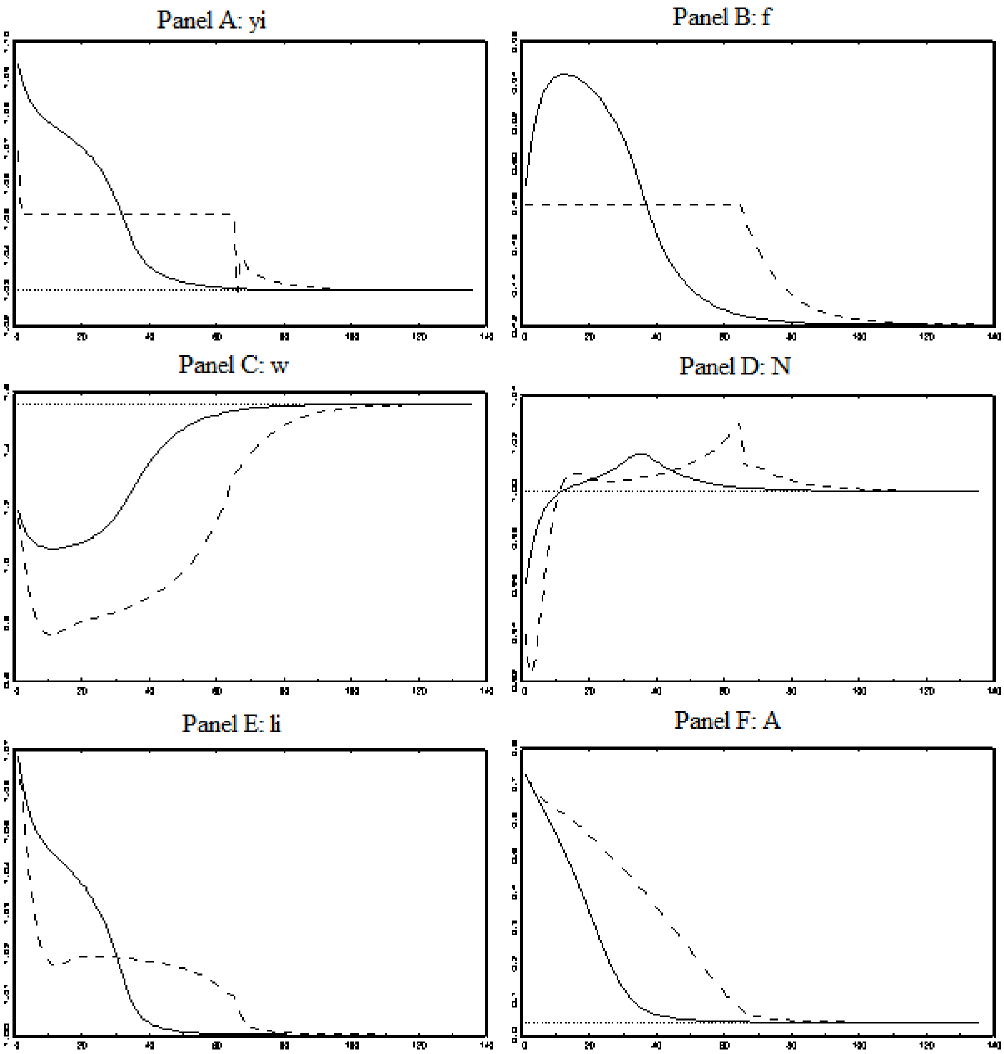

Figure 3: Urbanization Process with Distorted Choice of Technology: $f^* = 0.48$

still re‡ects the benchmark situation when ϵ = 0.1.

We …rst notice that ϵ does not impact the steady state of the economy so that the two lines in Figure 3 will eventually converge. Yet when ϵ is set to 0.05, the speed of migration is reduced, that is, l is lower at the begining (see the dashed line in Panel E). The slow migration makes the unemployment in cities less serious (see Panel D) and thus the wage is reduced less (see Panel C, note that we no longer present here the wage in rural sector). The relatively higher wage in urban sector will induce the …rm to adopt the production mode with relatively lower f (or less labor-intensive, see Panel B), and thus make investment less productive in creating capacity. This will lead to a relatively lower growth in output produced by the urban sector (see Panel A, notice that we no longer presents here with the growth rate of output produced by rural sector). Overall, the urbanization will proceed slowly since A declines slowly (see the dashed line in Panel F). We thus …nd that adding barrier to migration is not beni…t to the economy in terms of fast urbanization.

4.5 The productivity intergration

Next, we shall discuss the other policy parameter, that is, κ. As we have known, κ is related to how the agriculture economy is integrated into the aggregate economy in terms of economy-wide productivity (scienti…c knowledge, public infrastructure, etc.). It is thus also interesting to look at the urbanization process when we set κ di¤erently. In Figure 5 we add a simulation, re‡ected by the dashed line, of the urbanization process when κ = 0.8. The solid line still corresponds the banchmark case as re‡ected in Figure 2.

One …rst …nds that a di¤erent κ implies a di¤erent steady state of A , the proportion of people lives in rural sector. From Proposition 3, the higher is κ, the higher is A, the steady state of A . The other steady states are not impacted by κ.

Let us now look at the speed of urbanization process when κ sets di¤erently. The higher κ also indicates that the real income per unit e¢ ciency labor in rural sector is higher other things equal. Thus, the wage di¤erential will be smaller, less labor will be moved into the cities. All in all, the urban-

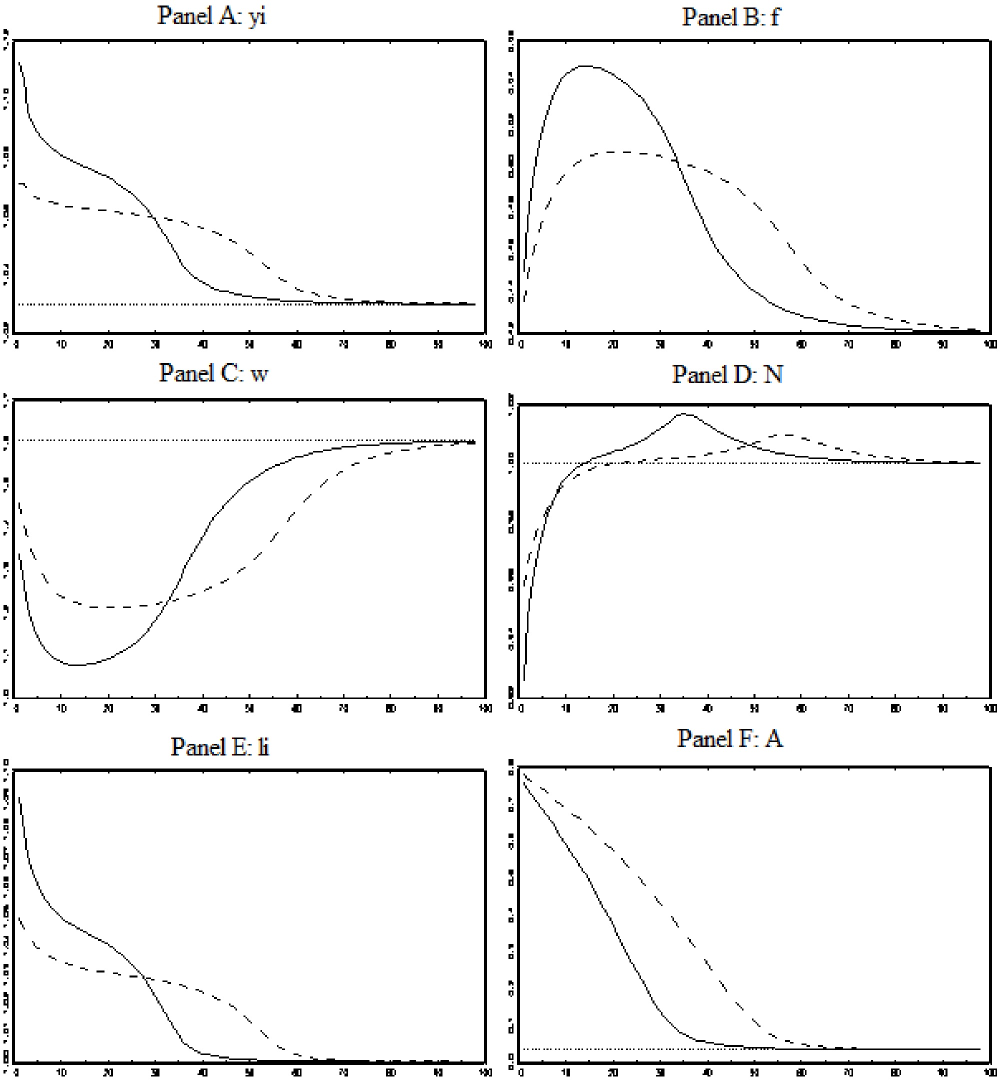

Figure 4: Urbanization Process with Di¤erent Barrier: Solid line for $\epsilon = 0.1$, dashed line for $\epsilon = 0.05$.

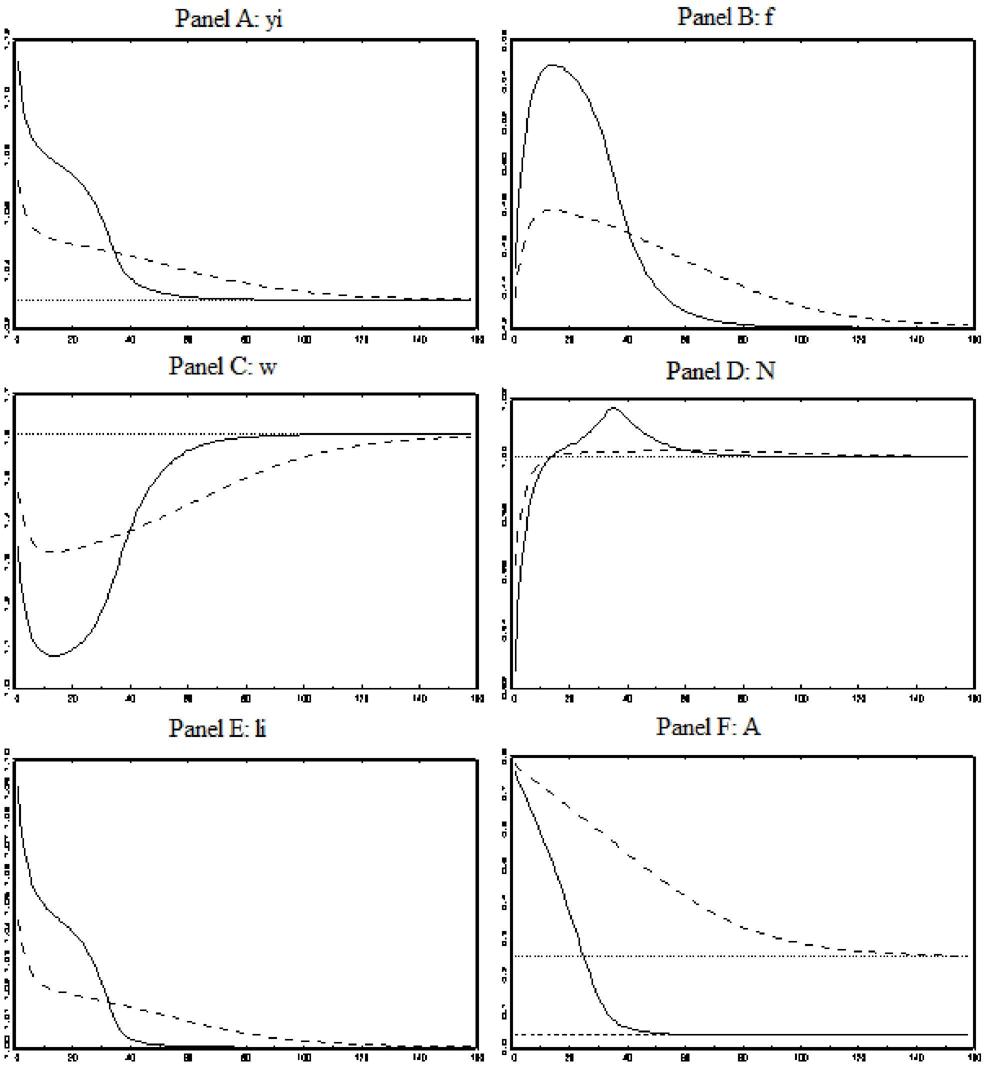

Figure 5: Unbanziation Process Under Di¤erent Producitity Intergrations: Solid for $\kappa = 0.5$, dashed line for $\kappa = 0.8$.

ization process will proceed slowly. This seems to suggest that it is perhaps not a good policy to intergrate the rural sector into the aggregate economy in terms of productivity at the earlier stage of urbanization.

4.6 The aggregate growth

The previous analysis only concerns the growth rate of output produced by urban sector, though we have also seen the growth in rural sector is declining during urbanization. How the urbanization process will impact the growth of aggregate output produced by rural and urban sector? To observe this, let us …rst establish the following proposition with regard to the gross growth rate of aggregate product.

Proposition 4 Let $Y \equiv Y \quad + Y$ denote the aggregate output in the economy. The gross growth rate of Y denoted as y can be written as

$$y = y \left(\frac{1}{1+\xi}\right) + y \left(\frac{\xi}{1+\xi}\right) \tag{30}$$

where y and y are given respectively in (22) and (26) and ξ follows

$$\xi = \left(\frac{A^{\partial}}{A_{-1}}\right) \left(\frac{w_{-2}}{w_{-1}}\right) \frac{x}{k}\xi_{-1} \qquad I \tag{31}$$

We shall remark that ξ in Proposition 4 is indeed the proportion of rural output over urban output: $\xi \equiv \frac{A,t}{I,t}$. Proposition 4 indicates that the dynamics of y, that is, the gross growth rate of aggregate output may heavily be impacted by the initial condition of ξ. Figure 6 provides the simulation to the urbanization process with di¤erent initial conditions of ξ.

In the …rst place, we shall remark either ξ and y has no feedback e¤ect to the other variables in the economy. Thus the growth rates of urban output (dashed line in Panel A, B, C) and rural output (dotted line in Panel A, B, C) are kept the same with di¤erent initial conditions ξ_0. Though these two growth rates are not changed, the growth rate of aggregate output does change signi…cantly with di¤erent ξ_0's. For a highly agricultured economy with higher initial condition ξ_0, urbanization may initially generates slow (or

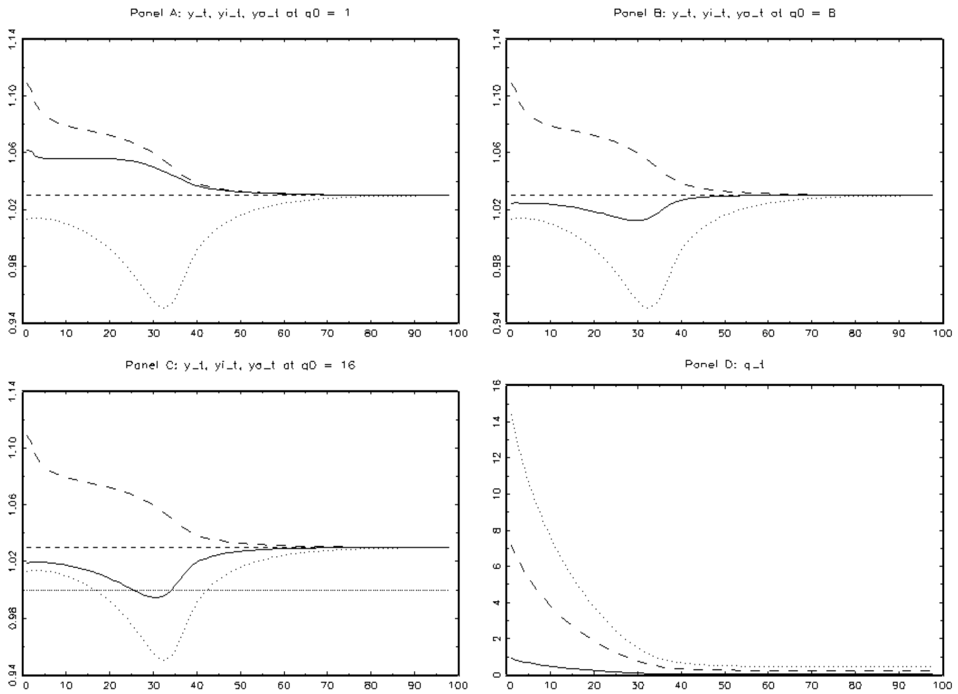

Figure 6: The Growth in Aggregate Output and Proportion of Rural to Urban Output

even negative) economic growth. The reason of this result may be expressed as follows: Once A_0, the population proportion has been given, an extremely high ξ_0 only indicates that the economy has so less capital stock. Thus, it is quite possible that the marginal product of labor in rural sector is larger than in urban sector.

In Panel D, we illustrate the dynamics of ξ (that is, the proportion of rural output to urban output) with di¤erent initial conditions: the solid line for ξ_0 = 1, the dashed line for ξ_0 = 8, and the dotted line for ξ_0 = 16. We …nd that history does matter here: a higher initial ξ_0 leads to a higher steady state of ξ . In our simulation, the steady state values of ξ in corresponding to the initial condition ξ_0 of being 1, 8 and 16 are respectively equal to 0.0264, 0.2113 and 0.4227.

5 Evidence, related literature and discussion

We shall now summarize what we have obtained from analyzing our two-sector model.

1. The growth rate of real GDP is generally higher (higher than the steady state growth rate) during the urbanization process, indicating the convergency hypothesis is satis…ed.

2. The real wage rate for e¢ ciency labor is generally lower (lower than the steady state of wage rate) during the urbanization process.

3. The production mode is generally more labor-intensive (comparing to the production mode at the steady state), indicating the principle of comparative advantage if the economy operates under market mechanism.

4. The distortion to the principle of comparative advantage (which may come from government intervention) will reduce the growth rate in real GDP and prolong the urbanization process.

5. Adding barrier to migration from rural to urban sector will reduce the growth rate in real GDP and prolong the urbanization process.

6. Intergrating rural economy into aggregate economy in terms of productivity (scientific knowledge, public infrastructure, etc.) too earlier may reduce the growth rate of real GDP and prolong urbanization process.

Next, we shall discuss the empirical evidence and the related literature that are consistent with what have found in the model.

The convergency hypothesis is well-established in academics and supported by the evidence from OECD countries (Barro and Sala-i-Martin, 1991, 1992). Since this hypothesis is only conditional (not all the less developed countries can be in the convergency path), the model we presented here is not a general model for less developed economies. It is the model for those less developed economy whose institutional structure are allowed them to be in the convergency path.

In Figure 7, we describe the data of real wage for efficiency labor w with different development level measured by GDP per capita. The data include 17 countries with the sample period from 1975 to 2007.[8] GDP per capita are measured by U.S. dollar at constant price of 2000. The wage for efficiency labor are measured as wage per hour in manufacturing divided by real GDP per hour worked, both converted to U.S. dollars at constant price of 2005.[9] The figure shows that the real wage in efficiency labor is increassing with the increase in the development level measured by GDP per capita. This is consistent with what we have found in the model as described by aforementioned point 2.

The principle of comparative advantage (as expressed in point 3 and 4) can be traced back to Richardo. This principle has been studied extensively in economics. Yet, controversial on this principle also exists either in academics and in empirics of development strategy. In China, the development strategy in 1950's is set as "the priority development of capital-intensive heavy indus-

[8] This 17 countries include Australia, Austria, Belgium, Canada, Denmark, France, Germany, Italy, Japan, Korea, Netherlands, Norway, Singapore, Spain, Sweden, United Kingdom, United States. These are the countries whose data are available for our international comparison.

[9] The data of hourly compensation (wage) and hourly contribution to GDP in manufacturing come from U.S. Department of Labor, Bureau of Labor Statistics, March 2009. The data of GDP per capita are from World Bank's WPI.

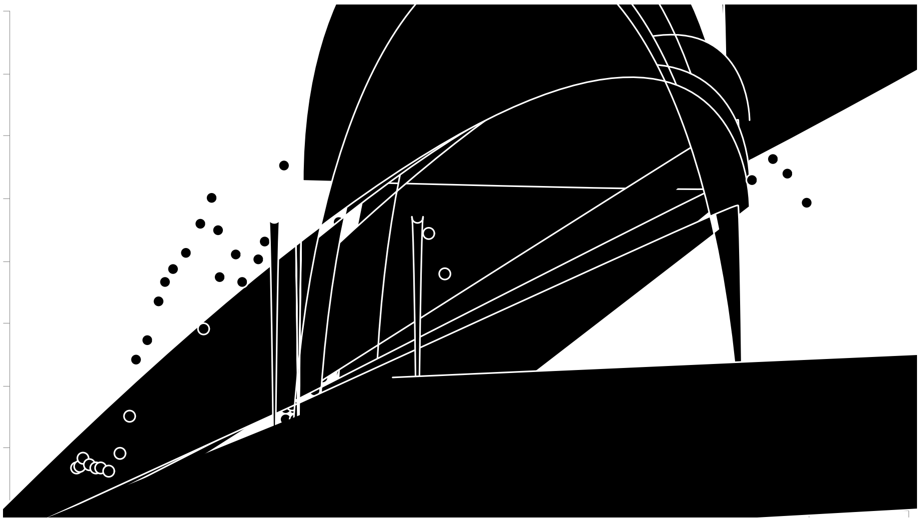

Figure 7: Wage in E¢ ciency Labor with Per Capita GDP

try."(Lin, 2007) The import-substitution stratragy that once dominated in academics of development economics and had been adoped by many countries is another example. The recent research on this principle including the related empirical studies can be found in Lin (2003), Lin and Liu (2004), Lin and Zhang (2007) and Lin (2007).

6 Appendix

6.1 The Proof of Proposition 1

Proof. By definition,

$$
\begin{aligned}
\sum_{=0}^{\infty}\beta\ FC_{\ +} = {} & r\ I\ \ + r_{\ -1}I_{\ -1} + r_{\ -2}I_{\ -2} + \cdots + r_{\ -}\ I_{\ -}\ + I_{\ -} + \\
& \beta\,(r_{\ +1}I_{\ +1} + r\ I\ \ + r_{\ -1}I_{\ -1} + \cdots + r_{\ -}\ I_{\ -\ +1} + I_{\ -\ +1}) + \\
& \beta^{2}\,(r_{\ +2}I_{\ +2} + r_{\ +1}I_{\ +1} + r\ I\ \ + \cdots + r_{\ -}\ I_{\ -\ +2} + I_{\ -\ +2}) + \\
& \vdots
\end{aligned}
$$

Including all financial costs resulting from $I_{\ -1}, I_{\ -2}, \ldots, I_{\ -}$ into the item of F , we find that the above equation can also be written as

$$
\begin{aligned}
\sum_{=0}^{\infty}\beta\ FC_{\ +} = {} & F\ \ + (1 + \beta + \beta^{2} + \cdots + \beta\ )r\ I\ \ + \beta\ I\ \ + \\
& \beta(1 + \beta + \beta^{2} + \cdots + \beta\ )r_{\ +1}I_{\ +1} + \beta^{\ +1}I_{\ +1} + \cdots + \\
& \beta\ (1 + \beta + \beta^{2} + \cdots + \beta\ )r_{\ +}\ I_{\ +}\ + \beta^{\ +}\ I_{\ +}\ + \cdots
\end{aligned}
$$

Thus, as long as τ is large enough, we obtain

$$
\begin{aligned}
\sum_{=0}^{\infty}\beta\ FC_{\ +} \approx {} & F\ \ + \frac{1}{1-\beta}r\ I\ \ + \frac{\beta}{1-\beta}r_{\ +1}I_{\ +1} + \cdots \\
& + \frac{\beta}{1-\beta}r_{\ +}\ I_{\ +}\ + \cdots
\end{aligned}
$$

Re-organizing the above equation allows us to obtain (8) as in Proposition 1. ■

6.2 The Proof of Proposition 2

Proof. Substituting (6), (8) and (10) into (9), the value of the objective function in (9) can be written as

$$
\begin{aligned}
L &= E\sum_{=0}^{\infty}\beta\left[Y_{\ +} \quad W_{+\ -1}\left(\frac{Y_{\ +}}{K_{\ +}}\right)^{\frac{1-\alpha}{\alpha}}\frac{Y_{\ +}}{X_{+}}\right] \\
&\quad F \quad -\frac{1}{1-\beta}E\sum_{=0}^{\infty}\beta\, r_{+}\,[K_{\ +} \quad (1-d)K_{\ +\ -1}]
\end{aligned}
$$

Setting $\frac{}{_{i,t+j}}$ = 0, then …rst-order condition can thus be expressed as:

$$
\frac{(1-\alpha)W_{+\ -1}Y_{\ +}}{\alpha X_{+}}\left(\frac{Y_{\ +}}{K_{\ +}}\right)^{\frac{1}{\alpha}-2}\frac{Y_{\ +}}{K^2_{\ +}} \quad \frac{r_{+}}{1-\beta} + \frac{r_{+\ +1}}{1-\beta}\beta(1-d) = 0
$$

Re-organizing the above equation allows us to obtain

$$
\frac{(1-\alpha)W_{+\ -1}^{\ \partial}}{\alpha X_{+}}\left(\frac{Y_{\ +}}{K_{\ +}}\right)^{1} \quad -\frac{r_{+}}{1-\beta} + \frac{r_{+\ +1}}{1-\beta}\beta(1-d) = 0
$$

Solving the above equation for $\frac{_{i,t+j}}{_{i,t+j}}$ while recognizing that $X_{+} = xX_{+\ -1}$ and $w_{+\ -1} = \frac{_{t+j-1}}{_{t+j-1}}$, we obtain (11) as in the proposition. ■

6.3 The Proof of Proposition 3

Proof. At the steady state, equation (19) can be written as

$$
\theta N + N(1-\theta)\left(N\right) = 1
$$

Since $\theta N + N(1-\theta)\left(N\right)$ is monotonically increasing in N, $N = 1$ is the unique solution to the above equation. Also at steady state, $w = w$ and thus from (29), $l = 1$. When $N = 1$ and $l = 1$, equation (24) evaluated at the steady state immediately allows us to obtain $y = x$. From equation (22), this further indicates that $k = x$. Given k, equation (23) allows us to obtain f as in the proposition. Given f, equation (20) allow us to obtain w.

At the steady state, equation (26) allows us to obtain $y = x$. Finally, given w, equation (27) allows us to obtain A when L is normalized at 1. ■

6.4 The Proof of Proposition 4

Proof. By de…nition,

$$y = \frac{y \quad Y_{\quad -1} + y \quad Y_{\quad -1}}{Y_{-1}} \tag{32}$$

where y and y are given respectively in (22) and (26). Note that

$$\frac{Y_{\quad -1}}{Y_{-1}} = \frac{Y_{\quad -1}}{Y_{\quad -1} + Y_{\quad -1}} = \frac{1}{1 + \xi_{\quad -1}} \tag{33}$$

where

$$\xi = \frac{Y}{Y} \tag{34}$$

In the same time

$$\frac{Y_{\quad -1}}{Y_{-1}} = 1 - \frac{1}{1 + \xi_{\quad -1}} = \frac{\xi_{\quad -1}}{1 + \xi_{\quad -1}} \tag{35}$$

Substituting (33) and (35) into (32), we obtain (30) as in the proposition. Expressing Y and Y in terms of (21) and (25), we obtain from (34):

$$\xi = \frac{\kappa X \ (A \ )}{f(w_{\quad -1})K} \qquad I$$

This equation also allows us to derive (31) as in the proposition. ■

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