Understanding the Puzzling Effects of Technology Shocks

Pengfei Wang
Hong Kong University of Science and Technology

Yi Wen
Federal Reserve Bank of St. Louis

Introduction

- RBC theory: technology expansionary.
- Gali (AER 1999) and Basu et al. (AER 2006): technology contractionary for I_t & N_t.
- Two implications: (i) technology shocks not main driving force; (ii) sticky prices.
- "the RBC theory is dead" (Francis and Ramey, JME 2005).

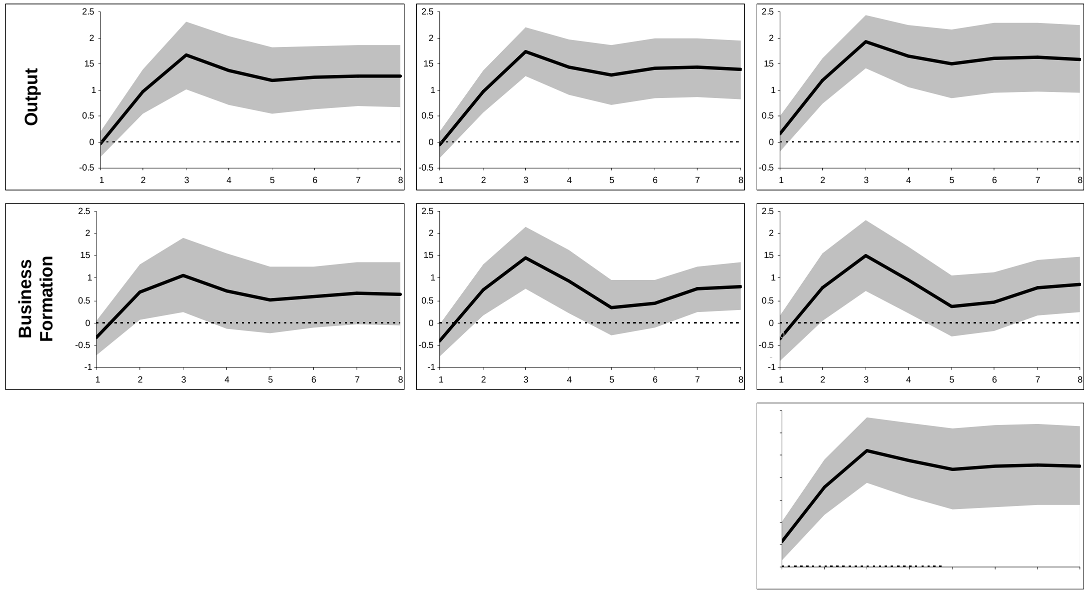
- It is possible that technology shocks not important and prices sticky.
- However, the finding of Gali and Basu et al. does not logically imply these are indeed the case.
- (i) the sign of the initial impulse responses to technology shocks does not imply lack of procyclicality.
- (ii) contractionary effect of technology shocks does not necessarily reject flexible prices – the main ficus of our paper.

- In what follows, we first present empirical regularities that appear to be profoundly inconsistent with flexible prices. Then we show that this is not the case.

Stylized Facts

$$\begin{vmatrix} 1 & 0 \\ -c_0 & 1 \end{vmatrix} \begin{vmatrix} x_t \\ y_t \end{vmatrix} = \begin{vmatrix} a_1 & 0 \\ c_1 & b_1 \end{vmatrix} \begin{vmatrix} x_{t-1} \\ y_{t-1} \end{vmatrix} + \begin{vmatrix} a_2 & 0 \\ c_2 & b_2 \end{vmatrix} \begin{vmatrix} x_{t-2} \\ y_{t-2} \end{vmatrix} + \begin{vmatrix} \varepsilon_t \\ v_t \end{vmatrix}$$

$$\begin{vmatrix} x_t \\ y_t \end{vmatrix} = \begin{vmatrix} a_1 & d_1 \\ c_1 & b_1 \end{vmatrix} \begin{vmatrix} x_{t-1} \\ y_{t-1} \end{vmatrix} + \begin{vmatrix} a_2 & d_2 \\ c_2 & b_2 \end{vmatrix} \begin{vmatrix} x_{t-2} \\ y_{t-2} \end{vmatrix} + \begin{vmatrix} e_{xt} \\ e_{yt} \end{vmatrix}$$


Output
Business Formation

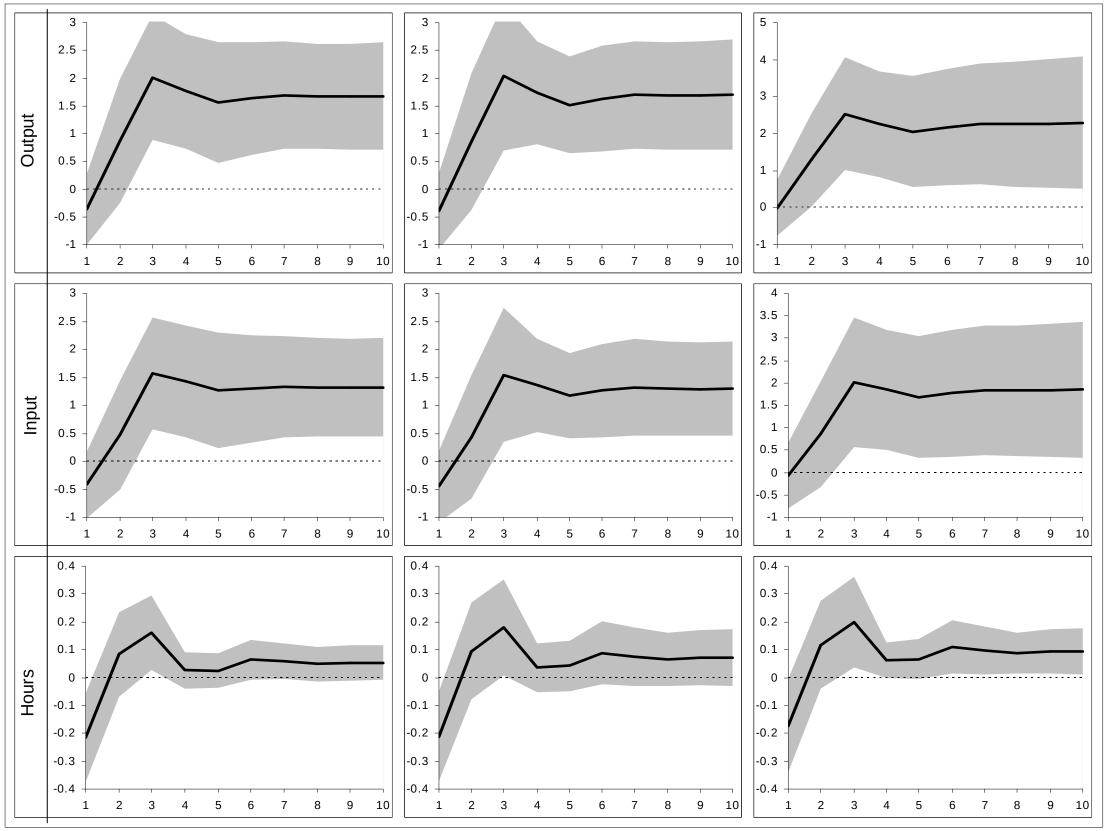


Figure 2. Sectorial Response to Agg. Tech. Shock

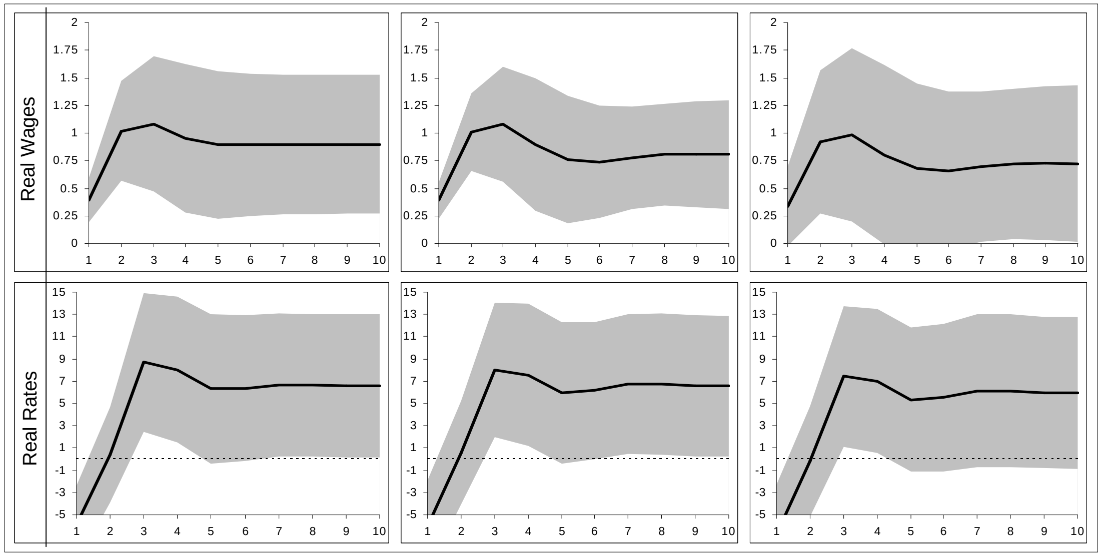


Figure 3. Response of Real Wage and Real Rate.

$$\Phi\alpha\frac{Y}{K} = r, \Phi(1-\alpha)\frac{Y}{N} = w.$$

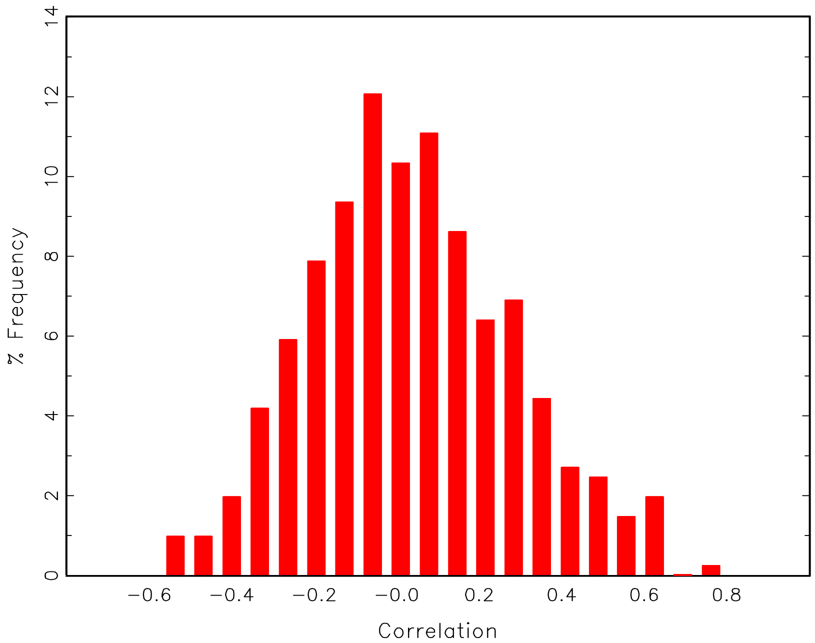


Figure 4. Distribution of Correlations

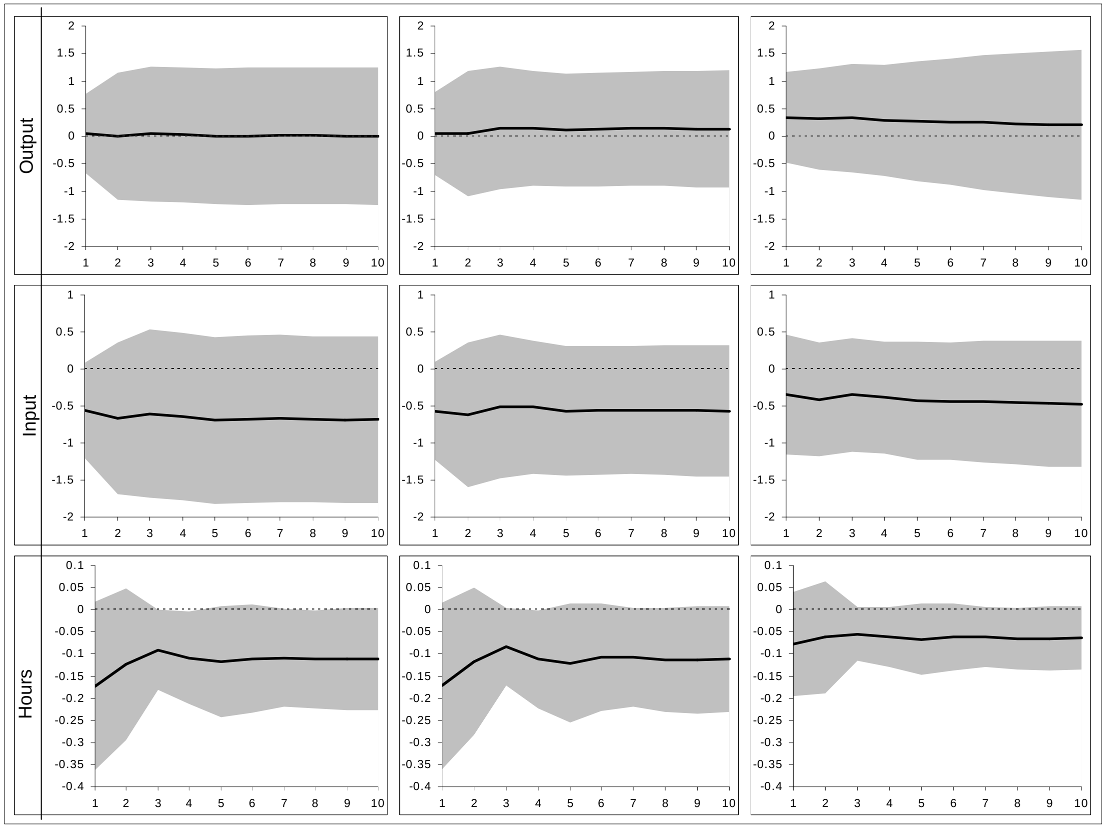


Fig 5. Sectorial Response to Sector-Specific

- Why tech shock contractionary and asymmetric?
- Our approach: Leontief technology at the firm level, with firm entry and exit. Prices fully flexible.
- Our model provides micro foundation to aggregate production functions, and is identical to a standard frictionless RBC model in aggregate dynamics if no time-to-build.
- However, with time-to-biuld, our model is able to explain all of the aforementioned empirical facts.

Benchmark Model

Final Good (y)

- Identical producers $i \in [0, \Omega_t]$, each producing one unit of final good. (Imagine a production assembly line with fixed production capacity.)

- Entry cost = Φ. Prob of exist = θ_t. Zero profit $\Rightarrow$ total number of producers Ω_t.

- Production function: $y = x$. Normalization: $p_y = 1$.

- Demand for input:

$$x = \begin{cases} 1 & \text{if } p_x \leq 1 \\ 0 & \text{if } p_x > 1 \end{cases}.$$

- Profit:

$$\pi = \begin{cases} 1 - p_x & \text{if } p_x \leq 1 \\ 0 & \text{if } p_x > 1 \end{cases}.$$

- Aggregate supply of output: $Y = \int_0^\Omega y di = \Omega$, aggregate demand for input is $\int_{i=0}^\Omega x di = \Omega$.

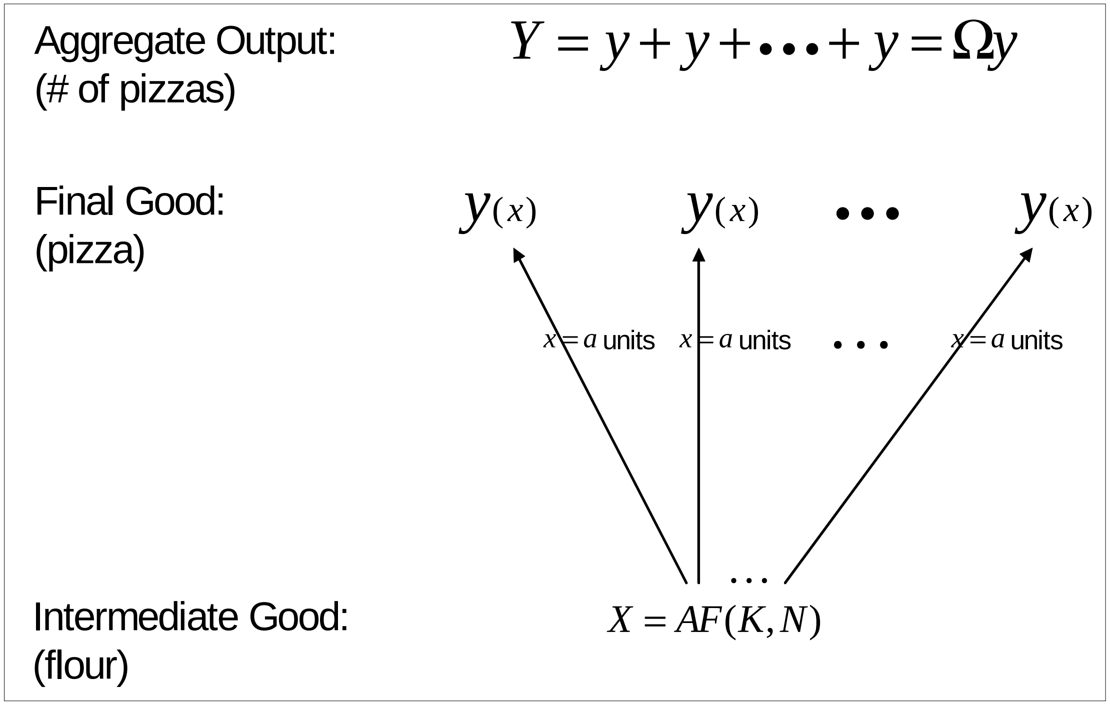


Figure 6. Production Structure.

- The value of a firm (with time-to-build):

$$V_t = \beta E_t \Lambda_{t+1} \pi_{t+1} + E_t \sum_{j=1}^{\infty} \beta^{j+1} \left[\prod_{i=1}^{j} (1 - \theta_{t+i}) \right] \Lambda_{t+j+1} \pi_{t+j+1},$$

$\Rightarrow$

$$V_t = \beta E_t \Lambda_{t+1} (\pi_{t+1} + (1 - \theta_{t+1}) V_{t+1}).$$

- Free entry $\Rightarrow V_t = \Phi$.

- Evolution of Ω:

$$\Omega_{t+1} = (1 - \theta_t)\Omega_t + s_t,$$

where s = new entrants.

Intermediate good

- Infinitely many identical intermediate good producers, with production function:

$$X_t = A_t K_t^{\alpha} N_t^{1-\alpha}.$$

- Profit maximization gives

$$\alpha p_x \frac{X}{K} = r_t + \delta,$$

$$(1-\alpha) p_x \frac{X}{N} = w_t.$$

- Perfect competition $\Rightarrow$ price equals marginal cost:

$$p_x = \frac{1}{A}\left(\frac{r+\delta}{\alpha}\right)^{\alpha}\left(\frac{w}{1-\alpha}\right)^{1-\alpha}.$$

- One representative firm $\rightarrow$ aggregate supply of intermediate good is X.

Household

- Net profit income (from final good producers):

$$\Pi_t = \int_{i=0}^{\Omega} \pi_t di - s_t \Phi.$$

- Utility maximization:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t [\log(C_t) + \gamma \log(1 - N_t)],$$

s.t.

$$C_t + K_{t+1} = w_t N_t + (1 + r_t) K_t + \Pi_t.$$

General equilibrium

$$\Phi = \beta E_t \Lambda_{t+1}(\pi_{t+1} + (1-\theta_{t+1})\Phi), \qquad 1$$

$$\Omega_{t+1} = (1-\theta_t)\Omega_t + s_t, \qquad 2$$

$$\pi_t = 1 - p_{xt}, \qquad 3$$

$$\alpha p_{xt}\frac{Y_t}{K_t} = r_t + \delta, \quad (1-\alpha)p_{xt}\frac{Y_t}{N_t} = w_t \qquad 4$$

$$w_t C_t^{-1} = \gamma(1-N_t)^{-1}, \qquad 5$$

$$C_t^{-1} = \beta E_t C_{t+1}^{-1}(1+r_{t+1}). \qquad 6$$

$$C_t + K_{t+1} - (1-\delta)K_t + s_t\Phi = A_t K_t^{\alpha} N_t^{1-\alpha}, \qquad 7$$

Equivalence to standard RBC model

- Suppose $\theta = 1$ and no time-to-build.

- Then $V_t = \pi_t = \Phi$. Hence $p_{xt} = 1 - \Phi$ and $s_t = \Omega_t$.

- The aggregate resource constraint becomes
$$C_t + K_{t+1} - (1-\delta)K_t = (1-\Phi)A_t K_t^{\alpha} N_t^{1-\alpha}.$$

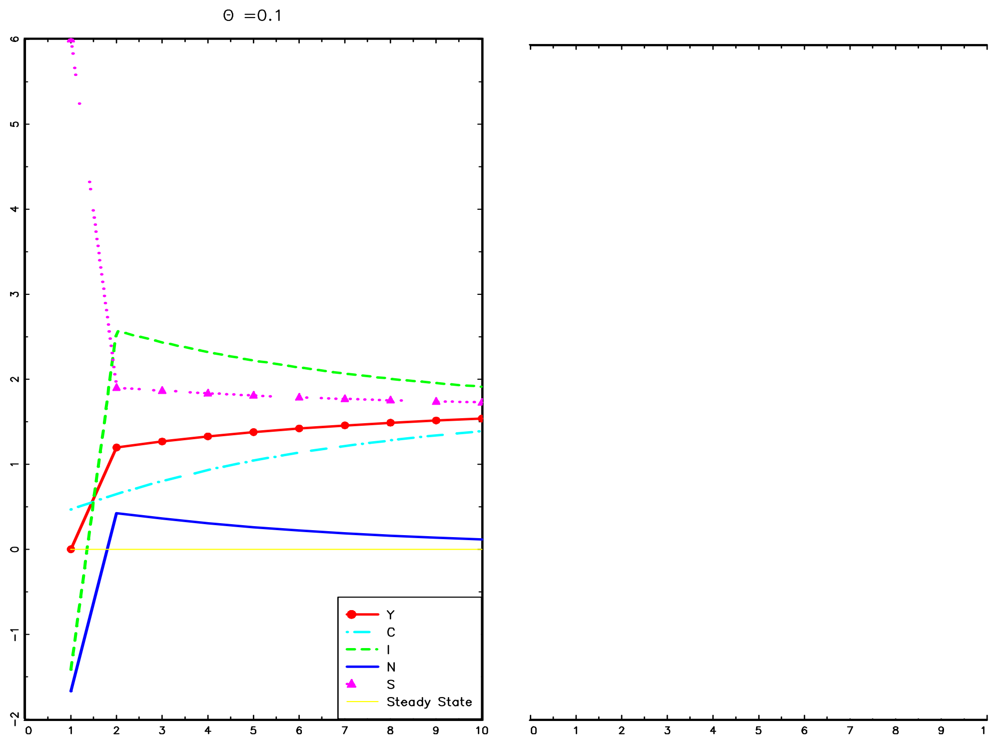
- The dynamics of this model are the same as those implied by a standard frictionless RBC model (e.g., King, Plosser and Rebelo, 1988).

Impulse responses

- *Calibration.* $\beta = 0.96, \alpha = 0.4, \delta = 0.1,\ \bar{N} = 0.2$ (about 35 hours per week). Let $\Phi = 0.1$. The results are not sensitive to these parameter values.

- Assume $\log(\theta_t) = \eta \log(\varepsilon_t)$. In the U.S. (1949-1996), 1% increase in ε reduces the business failure rate by 6%, hence we set $\eta = -6$.

- The average business failure rate (at annual frequency) for the U.S. economy implies $\bar{\theta} \approx 0.1$. We simulate the model using two alternative values, $\bar{\theta} = \{0.1, 0.25\}$. These values imply a steady-state markup in the range of $1.5 \sim 4\%$.


Θ =0.1
Y
C
I
N
S
Steady State

Multisector Model

- The production function:

$$y = \int_{j=0}^{1} x_j dj.$$

where the price of x_j is p_j.

- The demand for x_j:

$$x_j = \begin{cases} a_j & \text{if } p_j \leq 1 \\ 0 & \text{if } p_j > 1 \end{cases},$$

where $\langle a_j \rangle$ is the input-output coefficient matrix.

- The production function for intermediate good *j*:

$$X_j = AZ_j F(K_j, N_j).$$

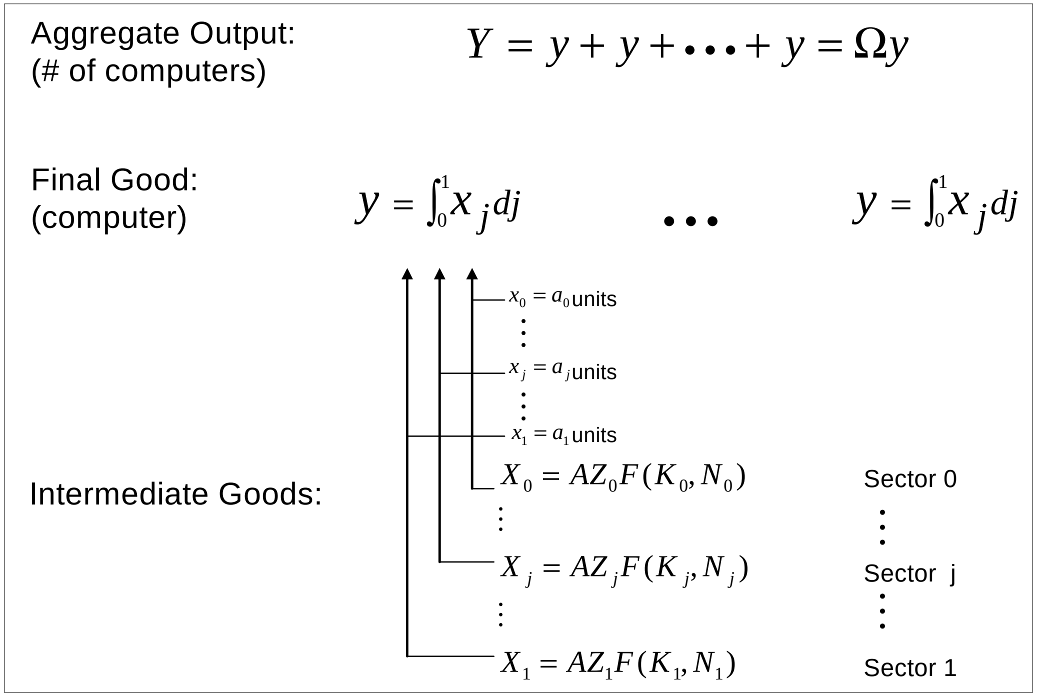


Figure 9. Multi-Sector Model.

- The gross profit for a final good producer is

$$\pi = y - \int_0^1 a_j p_j dj.$$

- The rest of the model's structure is similar:

  $V_t = \beta E_t \Lambda_{t+1}(\pi_{t+1} + (1-\theta_{t+1})V_{t+1}),\ \Omega_{t+1} = (1-\theta_t)\Omega_t + s_t,$
  $y = \int_{j=0}^{1} a_j dj = 1,\ Y_t = \int_{i=0}^{\Omega_t} y di = \Omega_t,\ \Pi = \int_{i=0}^{\Omega} \pi di - s\Phi,$
  $C_t + K_{t+1} = w_t N_t + (1+r_t)K_t + \Pi_t,$ where $K = \int_0^1 K_j dj$ and $N = \int_0^1 N_j dj$. The first order conditions for the household are the same as before.

- Profit maximization for each intermediate good firm in sector j gives $\alpha p_j \frac{X_j}{K_j} = r + \delta$ and $(1-\alpha)p_j \frac{X_j}{N_j} = w$. → Marginal cost of good j:

$$p_j = \frac{1}{AZ_j}\left(\frac{r+\delta}{\alpha}\right)^{\alpha}\left(\frac{w}{1-\alpha}\right)^{1-\alpha}.$$

- The aggregate output

$$Y = \int_{i=0}^{\Omega}\left(\int_{j=0}^{1} a_j dj\right)di = \int_{j=0}^{1}(a_j\Omega)dj,$$

where $a_j\Omega = X_j$ is the aggregate demand for intermediate good j.

- Hence, $\frac{X_j}{X_i} = \frac{a_j}{a_i}$, $\frac{Z_jK_j}{a_j} = \frac{Z_iK_i}{a_i}$ and $\frac{Z_jN_j}{a_j} = \frac{Z_iN_i}{a_i}$.

- $\rightarrow K_j = \left(\int_0^1 \frac{a_i}{Z_i}di\right)\frac{a_j}{Z_j}K$, $N_j = \left(\int_0^1 \frac{a_i}{Z_i}di\right)\frac{a_j}{Z_j}N$. Take the normalization, $\left(\int_0^1 \frac{a_i}{Z_i}di\right) = 1$, we have

$$K_j = \frac{a_j}{Z_j} K,$$

$$N_j = \frac{a_j}{Z_j} N.$$

- Substituting K_j and N_j into $X_j = A Z_j K_j^{\alpha} N_j^{1-\alpha}$ gives

$$X_j = a_j A K^{\alpha} N^{1-\alpha}.$$

- In equilibrium the final good production function becomes

$$Y = \int_{j=0}^{1} (a_j \Omega) dj = \int_{j=0}^{1} X_j dj = A K^{\alpha} N^{1-\alpha}.$$

Impulse responses

- Impulse responses of aggregate variables, such as $\{Y, C, I, N\}$, to aggregate technology shocks are the same as before.

- Impulse responses of sectors to aggregate and sector-specific technology shocks:

$$K_j = \frac{a_j}{Z_j} K,$$

$$N_j = \frac{a_j}{Z_j} N.$$

$$X_j = a_j Y.$$

- Equivalence to standard RBC model: Yes, if $\theta = 1$ and no time to build.

Explaining Heterogeneity

- Although our model is broadly consistent with stylized facts, it lacks the ability to explain heterogeneous responses across sectors.

- Consider final good firms are heterogenous because each firm i gets a different draw of a_j. Namely, firm i can transform one unit of intermediate good j into $a(i,j)$ units of final good. → Input-output matrix = $\{a(i,j)\}_{i\in[0,\Omega],j\in[0,1]}$.

- The production function:

$$y_i = \int_0^1 a(i,j)I(i,j)dj,$$

where $I(i,j) = 1$ if $a(i,j) \geq p_j$ and $I(i,j) = 0$ if $a(i,j) < p_j$.

- Assume $f_j(a_{i,j}) \neq f_k(a_{i,j})$ if $j \neq k$. Denote $F_j(p_j) = \Pr[a(i,j) \geq p_j] = \int_{p_j} a(i,j) f_j(a) da$.

- The aggregate demand for intermediate good j, by the law of large number, is then

$$X_{jf}$$

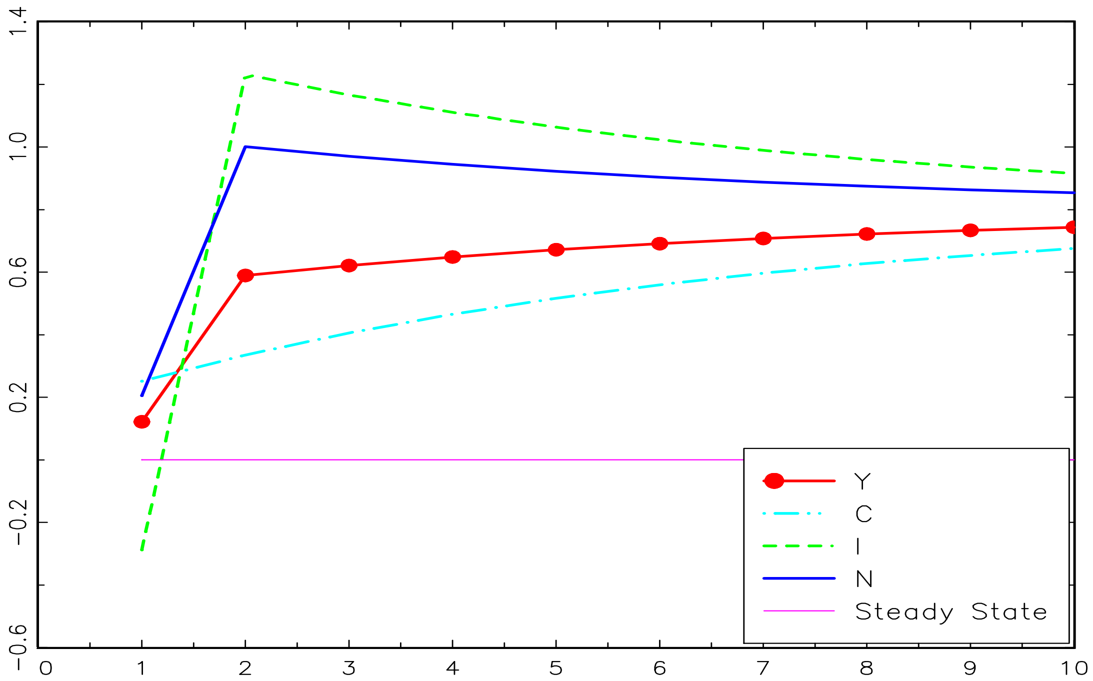
Impulse responses to sector-specific technology shocks.

- Around the steady state the percentage change of factor demand with respect to Z_j are given by

$$\hat{K}_j = (\epsilon_j - 1)\hat{Z}_j,$$

$$\hat{N}_j = (\epsilon_j - 1)\hat{Z}_j;$$

- Hence, allowing for heterogeneity in $f_j(a)$ can explain the heterogenous responses of inputs across sectors. This has little effects on the impulse responses of the model to aggregate technology shocks.



Responses to Demand.

Discussion

- A micro level rigidity in factor-demand does not by itself imply any aggregate rigidities, as long as Ω is variable.

- Example 1:

$$y_i = \int a_{i,j} I(i,j) di,$$

where $a_{i,j} \sim$ Pareto distribution $F(a) = 1 - (\frac{1}{a}$

$$y_i = a_i k + b_i n,$$

where k is capital, n is labor, and $\{a_i, b_i\} \sim$ Pareto distribution.

- Let the demand functions be

$$k = \alpha \text{ if } a_i \geq r, \text{ otherwise } k = 0;$$

$$n = \beta \text{ if } b_i \geq w, \text{ otherwise } n = 0;$$

where $\{r, w\}$ stand for prices of capital and labor.

- If $\theta = 1$ and no time-to-build, we obtain

$$Y = A(\Phi)\left[\alpha^{\frac{1}{\sigma}} K^{\frac{\sigma-1}{\sigma}} + \beta^{\frac{1}{\sigma}} L^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}.$$

- Example 3: If the Pareto distribution is replaced by the **Uniform** distribution, then

$$Y = \int_0^1 X_j dj - \left(\frac{\Phi}{2}\right)^{\frac{1}{2}} \left(\int_0^1 X_j^2 dj\right)^{\frac{1}{2}}.$$

- Example 4: Define production function

$$y_i = \int_0^1 h(a_{i,j}) I(i,j) dj,$$

where h is a truncated linear function satisfying

$$h(a) = \begin{cases} a & \text{if } a \leq a_{\max} \\ a_{\max} & \text{if } a > a_{\max} \end{cases}$$

, where $a_{\max} \in (1, \infty)$ is an arbitrary truncation point.

- Under Pareto distribution ($\sigma = 1$), we have

$$Y = \frac{(1+\Phi)a_{\max}}{\exp(\Phi)} \exp\left\{\int_0^1 \log(X_j) d_j\right\},$$

which is the Cobb-Douglas function with continuum of inputs.

- A special case:

$$y_i = h(a_i)k + h(b_i)n.$$

We have

$$Y = \tilde{B}(\Phi) K^{\frac{\alpha}{\alpha+\beta}} L^{\frac{\beta}{\alpha+\beta}}.$$