On the Faustian Dynamics of Policy and Political Power

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Revised June 12, 2009^{\ddagger}

Abstract

This paper examines the *Faustian dynamics* of policy and power. We posit a general class of dynamic games in which current policies affect the future distribution of political power, resulting in the following "Faustian trade off": if the current ruler chooses his preferred policy, he then sacrifices future political power; yet if he wants to preserve his future power, he must sacrifice his present policy objectives. The trade-off comes from the fact that the current political ruler/pivotal voter cannot un-couple the direct effect of his policy from its indirect effect on future power.

A *Policy-endogenous (PE) equilibrium* describes this endogenous transfer of power, and the resulting evolution of policy and political power over time. We show that the Faustian trade-off in a PE equilibrium is decomposed into two basic rationales. Thea4ic roiio68e2iumva1927(o)28-296(69013)-1ee2

1 Introduction

In Goethe's *Faust*, a well meaning Faust seeks knowledge, truth, and beauty, but cannot achieve them on his own. The devil appears and strikes a bargain with Faust: the devil will serve Faust while Faust remains here on earth; in exchange Faust must serve the devil in hell. But there is a catch. As part of the agreement, if Faust is so happy with the devil's services that he wants to "freeze" the present moment forever, Faust must then die immediately. Hence, the bargain endows Faust with the power to reach his objective but denies him the ability to savor it.¹

So it is, quite often, with "political bargains." If a political leader chooses a desirable but unpopular policy, he may lose political power and thus the ability to shape policy in the future. By opting to stay in power, he sacrifices his policy objectives and then faces the same trade-o in the future. Hence, the political bargain endows a leader with the power to determine policy only as long as he does not choose the one he prefers.

These types of "Faustian trade-o s" are not uncommon in politics. After signing the Civil Rights Act in 1964, President Lyndon Johnson remarked to an aid, "We have just lost the South for a generation," a fairly accurate forecast of Democratic losses to come.² Schonhardt-Bailey (2002, 2006) outlines a similar trade o faced by British Conservatives during the famous 19th century Parliamentary debate on the repeal of the Corn Laws.

"In May of 1846, a British Parliament consisting predominantly of landowners decided to forego protection for agriculture by repealing the famous Corn Laws, a decision that split the Conservative party for a generation... Within a month of gaining repeal, the ... Government fell and the Conservatives remained divided and for the most part out of o ce for decades to come." - Schonhardt-Bailey (2002, p.2).

The "Peelites" (followers of Conservative Prime Minister Sir Robert Peel) were aware of the political costs. Nevertheless, they justified their vote by appealing to the "necessity to preserve the territorial constitution."

This paper examines a dynamic game theoretic model of this "Faustian trade-o" between policy and power. To highlight the "tragic" nature of this trade o, we do not presume that

¹While there are many versions of the Faustian Bargain, the most well known is rendered by Goethe in (translated): *Faust: The Tragedy in Two Parts*, 1932 (translated in the original metres by Bayard Taylor). See also www.openlibrary.org/details/faustgoethe00goetiala

²See the following clickable link: LBJ in wikipedia.org. We thank a referee for pointing out the reference. Regarding the accuracy of LBJ's forecast, see clickable link: 1964 Civil Rights in uncommonliberty.blogspot.com.

rulers crave power for its own sake. Instead, political actors are assumed to be purely policymotivated. Their concern about losing power arises only because the new decision makers have di erent policy objectives than their own.

We model an ongoing society inhabited by a continuum of infinitely lived citizens. At each date t, one of the citizens, whom we refer to as the *leader* has e ective authority to choose a policy that a ects all the citizens in society. The "leader" can be viewed as an elected ruler, or alternatively, as a pivotal voter whose preferences are decisive in determining a policy. In either case, the policy choice of the leader in date t can change political power in a way that ultimately changes the identity of the leader in date t + 1. This can be done through policies that a ect the underlying demographic and/or distributional characteristics of the population. For instance, an increase in a country's income tax changes the future distribution of income. In turn, this may bring about electoral changes in future political power.

Because future political power is endogenously driven by current policy change, we refer to this as a case of *policy-endogenous (PE) political power*. Under policy-endogenous power, the dynamics induce a feedback loop from policy to power and back to policy. We characterize the *Policy-endogenous (PE) equilibria* — smooth Markov Perfect equilibria in policy-endogenous regimes.³ As a benchmark, PE equilibrium paths of policy and power are compared to those resulting from *permanent authority (PA) equilibria* — equilibria under which political power is permanent.

In the PE equilibrium, a political leader faces a *Faustian trade-off* when his most preferred policy strips him of power and then places it in the hands of a less desirable ruler. A central theme to emerge from the study is that Faustian trade o s tend to turn political decision makers into "Burkean conservatives." That is, when facing a Faustian trade o , a leader must overcompensate for the potential loss of power by slowing rate of political evolution as dictated by his current policy choice. This Burkean e ect is compounded by the indirect e ect one's decision has on the policies of future leaders: a fiscal conservative whose preferred might lead to an electoral loss to a fiscal liberal in the longer term. The result is that political shifts and policy changes are more gradual than they would be if these sorts of trade o s went unrecognized.

While an abundant literature in political economy studies the link from political power to policy, less is known about the "reverse causal link," i.e., from policy to power. One reason for this is that much of the "first generation" political economy literature studied

richer than another today is still expected to be richer after the policy change goes into e ect. Consequently, a fixed median voter emerges in equilibrium, and so no change in political power occurs.

The traditional emphasis on "undistorted" systems is a natural starting point. Yet, biases that produce policy-endogenous distortions have, historically, been the rule rather than the exception. Until the late 19th century, most governments explicitly weighted votes by some form of wealth or property value. In democracies today the bias is less formulaic but no less real. Representation in the U.S. Senate, for example, is biased in favor of less populous states, hence toward characteristics of rural rather than urban voters. Small minority parties in parliamentary systems often have disproportionate influence in the formation of majority governments. In addition, recent studies by Benabou (2000), Bartels (2004), and Campante (2007) all document the wealth-bias implicit in the U.S. political system.

By themselves, these biases may not create a Faustian trade o . However, policy choices typically have distributional e ects — changes in, say, income inequality — and we show that these e ects can create a Faustian trade o when coupled with the political bias.

Something akin to a Faustian trade o arises in some recent studies of endogenous electoral outcomes. These include Besley and Coate (1998), Bourguignon and Verdier (2000), Hassler, et. al. (2003), Dolmas and Hu man (2004), Ortega (2005), Hassler, et. al. (2005), Azzimonti (2005), Campante (2007), Acemoglu and Robinson (2008). Many of these focus on way in which particular political mechanisms such as Downsian competition a ect future voter preferences. These and other related papers are discussed in more detail in Section 5. We are not aware of a systemic study that identifies common features of the Faustian trade o (including the longer term indirect e ects mentioned above) across a large sweep of environments and political systems. This paper builds on the recent literature by working toward that end.

We develop the model in two stages. First, we posit a stylized model of public investment in which the level of investment augments a public capital good such as infrastructure or education. We then extend the results to a general (non-parametric) model. Each model features a "detail free" mechanism by which the policy-power link is specified in reduced form as a function from population characteristics to the type or identity of the leader. This mechanism is later "endogenized" by showing that any member of the class of reduced form functions considered here can be generated by a majority voting rule in which votes are weighted by wealth or income.

We analyze both transition dynamics and steady state properties of the model. In the stylized model, the Faustian trade o moderates each leader's incentives to invest in public capital. For instance, when public capital is below its equilibrium steady state level, each

Rodrik (1994), Krusell et. al. (1996, 1997) and Krusell and Rios-Rull (1999). For a detailed review of papers in this tradition, see Krusell et al (1997) and Persson and Tabellini (2000).

leader chooses a lower level of government investment than he would if his authority were permanent. Likewise, starting above the steady state investment, a leader chooses a larger investment than if his power were permanent. As a consequence, public capital initially changes more slowly than it would in the absence of a Faustian trade o .

We show that the preservation e ect on a given individual's incentives increases in magnitude through time. Yet, the preservation e ect is partly o set by a second rationale, the "reformation e ect," which induces more radical policies that hasten the rate of political evolution. The reformation e ect comes from the complementarity between current and future policies. More aggressive policies put future authority in the hands of even more aggressive types of leaders thus changing the marginal productivity of policy decisions in the present.

Overall, political power evolves toward more progressive leaders, and, in fact, the policy trajectory is more progressive than under permanent authority in the long run. The steady state stock is larger and the leader more progressive, than in a case of no Faustian trade o s.

The general model is introduced in Section 2. Section 3 elaborates on the parametric model. The model displays some of the salient features of political systems that gives rise to policy-endogenous power. Section 4 returns to the abstract model and contains the main decomposition result. Section 5 examines the related literature and examples. Finally, Section 6 discusses various extensions. The proofs are contained in an Appendix at the end.

2 The Basic Setup

In this Section, we describe a general model in which Faustian tradeo s occur. The level of generality underscores the fact that policy endogenous political change is not necessarily an isolated feature of a small set of environments. However, for concreteness a special case of the general model is presented in Section 3 in the form of a stylized model of public sector investment and growth.

2.1 The Environment

At each date t = 0, 1, 2, ... a government must undertake a policy decision that a ects all members of society. The policy interacts with a state variable that fully summarizes the economy at that date. Let a_t denote the policy choice and t the state. Feasible policies and states are restricted to compact intervals.

The state is assumed to evolve according to a deterministic Markov process determining the future state as a function of current states and actions.⁶ Formally, let $_{t+1} = Q(_t, a_t)$. The transition function Q is assumed smooth, nondecreasing in $_t$, increasing in a_t , and jointly concave in the pair $_t$ and a_t .

⁶An earlier version of the paper introduced the model with additive stochastic shocks of the form $\omega_{t+1} = \nu_{t+1} + Q(\omega_t, a_t)$ with $\{\nu_t\}$ iid across states and across time. We took them out to simplify notation since they added nothing to the qualitative features of the model.

Society is comprised of continuum I = [0, 1] of infinitely lived *citizen-types*. Given any sequence of states $\{ t \}$ and policies $\{a_t\}$, the dynamic payo to citizen-type $i \in I$ is

$$\sum_{t=0}^{\infty} {}^{t} u(i, t, a_t)$$
(1)

where is a common discount factor, and the payo function u is smooth, increasing in $_t$, decreasing and strictly concave in a_t , and jointly concave in the pair $_t$ and a_t .

The assumptions on u and Q reflect the idea that the policy a_t is a tax or public investment that, while costly in the present, augments the future value of the state. In turn, the state t is a parameter that determines a capital stock or income distribution. An obvious example is an income tax that generates revenue to fund public infrastructure.

2.2 The Permanent Authority Benchmark

Consider a benchmark case of a decision maker who faces no Faustian trade-o . There are a few ways this can happen. For instance, if all individuals have identical preferences over policy, then a purely policy-motivated type is indi erent between retaining and losing political power. Alternatively, an individual can lose power for purely exogenous reasons unrelated to his current policy choice.

The most natural benchmark, however, is that of an individual who maintains political power regardless of his policy action. This "king" or "dictator," whom we label i_0 , chooses a policy a_t at each date t, fully anticipating that his authority is perpetual. We refer to this as the *permanent authority* (*PA*) regime. PA regimes are not, almost by definition, common in modern democracies. They were common, however, in many European monarchies prior to the 20th century.

The PA regime can also be interpreted normatively as a time-consistent social planner. In our model, there is no qualitative di erence between the permanent authority of a fixed citizen-type, and the authority of a social planner who aggregates payo s across citizen-types. Significantly, most political economy models either assume explicitly a PA regime (e.g., a social planner) or derive one in equilibrium under special assumptions on preferences, technology, and political institutions.

Consider the problem of a permanent authority i_0 . His policy choices are characterized by a policy function $(t_t) = a_t$ (omitting the notational dependence on i_0) that solves the Bellman equation

$$V(i_0, t;) = \max_{a_t} [U(i_0, t, a_t) + V(i_0, t+1;)]$$
(2)

subject to $_{t+1} = Q(_{t}, a_t)$. We refer to the function that solves (2) as a *Permanent* Authority (PA) equilibrium. The PA equilibrium serves as a baseline for comparison.⁷

2.3 Policy-Endogenous Political Power

We compare the PA regime in which there are no Faustian trade o s to one in which there are. In an environment with *policy endogenous* (*PE*) *political power*, the current policy choices induce changes in future political power. To focus on decision-theoretic aspects, political power is modeled in reduced form by assuming that each period the political system produces an outcome that is rationalized by the preferences of a pivotal individual. This individual (e.g., pivotal voter or political leader) is, in e ect, endowed with the exclusive right in period t to choose the policy action a_t . The assumption that the political process admits a pivotal leader clearly involves some loss of generality. There are, however, commonly known conditions on policy preferences, notably the class of intermediate preferences (Grandmont (1978)), and preferences satisfying single crossing properties (Gans and Smart (1996) and Rothstein (1990)) that do admit pivotal voters.

Henceforth, we refer to the pivotal decision maker as the *leader*. Political power is therefore represented by a mapping from states (e.g., capital stocks, income distributions) to citizentypes. Formally, the mapping is assumed to be a smooth, weakly monotone function $\mu : \mapsto i$ such that $i_t = \mu(t_t)$ is the leader who decides on policy in state t_t .

Because μ determines "who's in charge" in each state, we refer to it as the *authority function*. The key attribute of an authority function, for our purposes, is that it admits the possibility that political power changes endogenously due to changes in the state. Current policy changes produce changes in the state which, in turn, produce changes in the identity of the leader through μ .

To facilitate a comparison with the PA regime, we fix i_0 as both the permanent authority and the *initial* decision maker under policy-endogenous power. The change in the identity of the leader, as described by μ , defines a dynamic game with a potentially infinite set of players. The players' choices result in a stationary Markov process that realizes states $\{0, 1, 2, ..., \}$, leaders $\{i_0, i_1, i_2, ..., \}$, and policies $\{a_0, a_1, a_2, ...\}$. We refer to these processes as the *Faustian dynamics* of PE political power.

In much of the paper, μ is treated as exogenous, although we give an explicit micro foundation for it in Section 3. In the simplest case of majority voting, $\mu(t)$ is the median type

⁷Implicitly, the PA equilibrium characterizes a time consistent optimal strategy for i_0 if the private sector's dynamic response is Markov or is absent altogether. In the case where the private sector is absent, the PA equilibrium amounts to a single agent dynamic programming problem, in which case it coincides with the full commitment solution.

whenever the Median Voter Theorem holds. However, in order to obtain a state-dependent authority function μ , votes in some cases may need to be weighted by, say, wealth or income. The idea roughly is that changes in the state "distort" the income distribution, and wealth-weighted voting is sensitive to these types of distortions — see Section 3 for an explicit

if and only if for all t_i , and for all a_{t_i} ,

$$V(\mu(_{t}), _{t}; ^{*}) \geq U(\mu(_{t}), _{t}, a_{t}) + V(\mu(_{t}), Q(_{t}, a_{t}); ^{*})$$
(4)

Clearly, there are other types of Subgame Perfect equilibria one might examine in a Faustian model. However, we think the restriction to Markov Perfection is sensible in this context. First, it facilitates a direct comparison with the permanent authority equilibrium in (2). Second, the restriction seems natural in large populations. Costs of coordination are presumably higher in larger economies, and so strategies that therefore depend only on the current, payo relevant state may reduce these costs. Third, if policy-makers have uniformly bounded recall then it can be shown that *any* Subgame Perfect equilibrium in our model *must* be Markov.¹⁰

Given a PE equilibrium *, consider what policy *would have been chosen* in state $_t$ by an arbitrary citizen-type *i*? This question is hypothetical because an arbitrary *i* is not the authority in state $_t$ unless it happens that $i = i_t = \mu(_t)$. The question is important nevertheless because it allows one to compare the Faustian incentives for di erent political types in any situation. Let $a_t = (i, _t)$ denote the hypothetical decision of type *i*. Call a *hypothetical equilibrium* if for all $_t$, all *i* and all a_t

 $U(i, t, (i, t)) + V(i, Q(t, (i, t)); *) \geq U(i, t, a_t) + V(i, Q(t, a_t); *)$ (5)

The inequality in (5) coincides with (4) in those states for which $i = \mu(t)$. The hypothetical equilibrium and the PE equilibrium are therefore related by $(\mu(t), t) = t^*(t)$.

Hence, starting from initial state $_0$, the decision maker i_0 chooses $(i_0, _0) \equiv *(_0)$. Type i_0 correctly anticipates that his chosen policy a_0 leads to a possible change in decision authority at date t = 1. This change is given by $i_1 = \mu(_1)$ where next period's state $_1$ is determined by $_1 = Q(_0, *(_0))$. It is generally true that $(i_0, _1) \neq *(_1)$ because i_0 no longer makes the decision in state $_1$ in the PE equilibrium, and the decision type i_1 will generally have di erent preferences over policy. The more interesting comparison, however, is between $(i_0, _1)$ and $(_1)$. It tells us how an arbitrary citizen-type would react to the loss of power in a given state, where the extent of the loss is determined by that state.

In certain instances, the current leader faces no Faustian trade-o even when μ is distortionary. For instance, suppose that period payo s are of the form $u(i, , a) = u_1(i) + u_2(, a)$ or the form $u_1(i)u_2(, a)$. In either case, the additive or multiplicative separability implies that individual-specific characteristics do not a ect policy preferences. Individuals' policy preferences are therefore the same, and so changes in power have no e ect on policy.

 $^{^{10}}$ An economy has uniformly bounded recall if there is a finite bound *m* such that every decision maker's memory of the past history cannot exceed *m* periods back. The argument for the stated assertion relies on asynchronous decision making, which applies to the present model. A simple proof is found in Bhaskar and Vega-Redondo (2002).

3 A Stylized Model of Public Sector Investment

This Section examines how Faustian dynamics work in a concrete, special case of the general model. We examine a stylized model of public sector investment where political authority is derived explicitly from a system of weighted voting. We examine two cases. One where the public sector expands monotonically as political power steadily evolves toward more fiscally liberal types. The other where public sector alternately expands and contracts and political power oscillates between fiscally liberal and conservative types of decision makers. We use the model to illustrate e ects of Faustian dynamics on the evolution of political power, public sector growth, and income inequality.

3.1 The Environment

Society invests each period in a public capital good (infrastructure) which is produced according to the linear transition

$$_{t+1} = Q(_{t}, a_t) = (1 - d)_{t} + a_t.$$
 (6)

Here, t is the current stock of public capital, a_t is public investment, and $d \in (0, 1]$ is the depreciation rate. Public investment a_t is produced from a lump sum tax \mathcal{T}_t , according to a concave production technology $a_t = (2\mathcal{T}_t)^{1/2}$. This technology can be alternatively expressed as the cost $\mathcal{T}_t = \frac{1}{2}a_t^2$ of providing public investment a_t .

Each citizen is assumed to provide labor inelastically with a time allocation normalized to one. His labor is combined with public capital to produce income

$$y(i, t) = g(i) + f(i) t$$

with g(i) and f(i) denoting type *i*'s e ciency utilization of labor and capital, respectively. We assume that $f' \ge 0$, so that higher types have (weakly) higher e ciency utilization of public capital.

A citizen's flow payo is assumed to be linear in his consumption, and we assume (without loss of generality, due to the linear payo) that there is no private borrowing or saving. A citizen therefore consumes his after-tax income $y(i, t) - T_t$. His flow payo is then given by

$$U = y(i, t) - T_t = g(i) + f(i) t - \frac{a^2}{2}$$

Dropping the labor e ciency term g(i) yields an indirect utility function of the form in (1), given by

$$U(i, t, a_t) = f(i) t - \frac{a_t^2}{2}.$$
 (7)

Notice that the idiosyncratic labor productivity g(i) drops out of (7) as it plays no role in citizen *i*'s policy preference. However, g(i) will play an indirect role in the evolution of the economy through its e ect on the political process.

3.2 A Voting Foundation for μ

The stylized model has a natural political interpretation. The state may be regarded as a proxy for the size of government. Hence, the assumption $f' \ge 0$ implies that the dynamic policy preferences are well ordered: higher types are more "fiscally liberal" in the sense that they prefer a larger public sector.¹¹ Given preferences satisfying (7), one can show that the Median Voter Theorem applies. Hence, any voting system, regardless of the way in which votes are weighted admits a pivotal voter whose preferred policy choice beats any other in a pairwise comparison.

One such voting system is that of wealth-weighted voting discussed in Section 2. Specifically, consider a polity that allocates y(i, t) votes to each type *i* in state t^{12} . Then, applying the Median Voter Theorem (where "median" is wealth-weighted here), the authority function $\mu(t)$ is endogenously determined by

$$\frac{\int_{0}^{\mu(\omega_{t})} y(i, t) di}{\int_{0}^{1} y(i, t) di} = \frac{\int_{0}^{\mu(\omega_{t})} [g(i) + f(i)] di}{\int_{0}^{1} [g(i) + f(i)] di} = \frac{1}{2}$$
(8)

Hence, while order-preserving shifts in the wealth distribution do not change the median voter, they do change the wealth-weighted pivotal voter. In this case, μ increases (decreases) whenever Lorenz inequality increases (decreases) in the state t.

An authority function μ can therefore be computed from (8) given functions f and g. A particularly tractable form of μ , given by

$$i_t = \mu(t_t) = \frac{0+t_t}{0+t_t+1}$$

with parameters and $_0 > 0$, may be obtained from a marginal e ciency f satisfying: $f(i) = \frac{i}{1-i}$ for all $i \le \overline{i}$, and $f(i) = f(\overline{i})$ for all $i \ge \overline{i}$. The marginal labor e ciency g can easily be backed out using (8). The details are contained in a technical appendix.¹³

$$V(i, \omega_t; \Psi^*) = f(i) V^1(\omega_t; \Psi^*) + V^2(\omega_t; \Psi^*).$$

¹¹To see this, observe that when flow payoffs satisfy (7), the value function is an affine function of f(i) regardless of the policy function Ψ^* :

 $^{^{12}}$ In general, we could have considered any weighting system that places positive weight on one's income/wealth. See Footnote 8 for real world examples.

¹³See www9.georgetown.edu/faculty/lagunofr/onlineappendix.pdf.

For the purposes of solving the model, the most relevant property of this construction is that for all $_{t}$,

$$f(\mu(t)) = 0 + t \tag{9}$$

In other words, authority is given to the type i_t for whom the marginal value of public sector capital is an a ne function of the stock itself. Increases in the stock therefore correspond to more fiscally liberal authorities (for > 0). By varying parameters and $_0$ such that all authority functions intersect the initial point ($_0$, i_0), Equation (9) maps out a one-dimensional *class* of authority functions, each of which di ers by the adjustment speed and direction of political power.¹⁴

Figure 1 illustrates three examples of authority functions that satisfy (9). In the Figure, μ° , μ' , and μ'' correspond to parameter values for = 0, ', ", respectively, where 0 < ' < ". The special case of = 0 corresponds to the "undistorted" case of permanent authority, with i_0 as the permanent leader. Clearly there is no Faustian trade-o for i_0 in this case. When = 0, it is not hard to show that the PA equilibrium is state-invariant: $(t_1) = 1.5^{15}$

The value | | measures the degree of institutional distortion. An increase in | | (while adjusting $_0$ to keep the same initial point) implies faster structural evolution of political authority. In the Figure, μ' and μ'' both begin with i_0 as the initial decision maker. The function μ' is "less distorted" than μ'' in the sense that μ'' o ers a starker Faustian trade-o for leader i_0 . Intuitively, one might call μ' more "conservative" in the "Burkean" sense of o ering a more gradual structural evolution of political power; μ'' is more "progressive" in the sense of inducing accelerated change. A central feature of the Faustian model, as we later show, is that equilibrium responses of individuals may undercut this seemingly straightforward comparison: the more conservative rule does not always produce a more gradual evolution of political authority in equilibrium.

In either case, > 0 implies that μ is increasing. An increase in public sector capital therefore adjusts political power upward toward more fiscally liberal types — those with higher marginal value of government expenditure. But more liberal types choose higher levels of government expenditure which increase the public capital stock. In this sense, μ represents a *reinforcing distortion*. If instead, < 0, then an increase in public capital would adjust political power downward, toward the more conservative "small government" types. In that

$$f\left(i_{0}\right)=\int h\left(i\right)f\left(i\right)di.$$

In other words, the social welfare function coincides with the utility function of a specially chosen i_0 . Therefore, the social planner's problem is the same as the PA problem with permanent authority vested in type i_0 .

¹⁴This class of authority functions are those that satisfy $\mu(\omega_t) = f^{-1}(\kappa_0 + \kappa \omega_t)$ such that κ and κ_0 satisfy the linear equation $f(i_0) = \kappa_0 + \kappa \omega_0$.

¹⁵ Because f(i) is a monotone function, the Permanent Authority equilibrium can be identified as a solution to a social planner's problem. To see this, suppose that h is a density on [0, 1] in which h(i) is the welfare weight assigned to citizen-type i. Then there exists i_0 such that

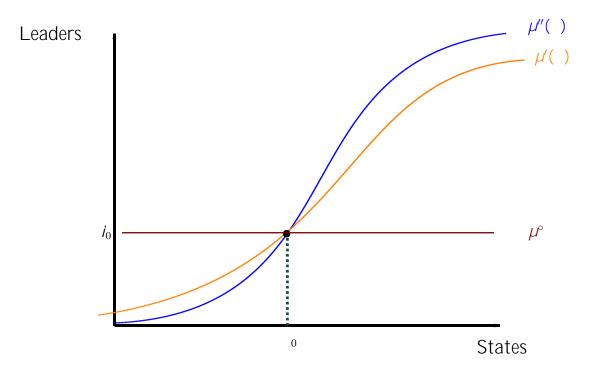


Figure 1: Authority Functions in the Policy-Endogenous and Permanent Power Regimes

case, because the authority function μ moves in opposition to the transition technology, it represents a *countervailing distortion*.

3.3 Monotone Faustian Dynamics

Consider first the case of > 0, thus a reinforcing distortion. Since μ slopes upward, increases in public capital put power in the hands of more fiscally liberal leaders. In the following result, refers to the state-invariant Permanent Authority (PA) equilibrium.

Proposition 1 Consider any authority function μ satisfying (9). If $0 < \langle d(\frac{1}{\delta} - 1 + d)$, then the following hold.

- (i) There exist an increasing, affine Policy-Endogenous (PE) equilibrium policy rule $*(_t)$ and a nonincreasing, affine Hypothetical rule $(i_0, _t)$, each of which are unique in the corresponding class of affine equilibria.
- (ii) The PE equilibrium path of states $\{ t \}$ converges monotonically to a unique steady state * = Q(*, *(*)). If 0 < *, then 0 < t implies $(i_0, t) < (t_1)$ and $(i_0, t_2) < (t_2)$.

*(t). Whereas if $_{0} > *$, then $_{t} < _{0}$ implies ($i_{0}, _{t}$) > (t) and ($i_{0}, _{t}$) > *(t).

(iii) If $_0 < *$, then there exists a state $\hat{}$ with $_0 < \hat{} < *$ such that

*($_{t}$)	<	$\begin{pmatrix} t \end{pmatrix}$	$\forall 0 \leq t < t$
*(_t)	>	$\begin{pmatrix} t \end{pmatrix}$	$\forall t > $

whereas if $_0 > *$, then there exists a state $\hat{}$ with $* < \hat{} < _0$ such that

* (_t)	>	$\begin{pmatrix} t \end{pmatrix}$	$\forall \hat{} < t \leq 0$
* (_t)	<	$\begin{pmatrix} t \end{pmatrix}$	$_t$ < $$

The upper bound on is required to satisfy a transversality condition that bounds the rate of growth. The Proof in the Appendix shows that the policy function of the a ne form $*(_t) = (d - K) * + K_t$ where K is a constant (in $_t$) with $0 \le K < d$, and * is the unique steady state. The coe cients implicitly vary in the exogenous parameters $_0$, , , and depreciation rate d.

Parts (ii) and (iii) state what will turn to be quite general properties of Faustian dynamics. Part (ii) compares the Hypothetical rule to both the PE and PA rules. If the initial state is below the steady state, then political power evolves from fiscally conservative toward more fiscally liberal types. By choosing a smaller expansion in government expenditures than he would if his power were permanent, type i_0 slows the rate of political change as it evolves away from his type. In this sense fiscal conservatism coincides with "Burkean" conservatism.

By contrast, when the initial state is above the steady state, then political power evolves from fiscally liberal toward fiscally conservative types. In that case, type i_0 , a fiscal liberal, acts as a Burkean conservative by choosing a larger expansion in government expenditures than he would if his power were permanent. By doing so, he once again slows the rate of political change.

For purposes of intuition, we restrict our discussion and pictures to the case of $_0 < *$. Then the (hypothetical) policies of leader i_0 become more conservative over time/states because more distant types assume power as the state progresses upward. But, as Part (iii) shows, individual caution is juxtaposed with progressive evolution of policy. The Policy-Endogenous equilibrium starts out more conservative than Permanent Authority, but winds up more fiscally liberal in long run. The intuition is as follows. Consider the incentives of the initial leader i_0 . He anticipates a growing economy under *. However, i_0 also anticipates the corresponding shift of power to more liberal types in the future. Because the higher tax rates chosen by these liberal types are undesirable from i_0 's viewpoint, i_0 slows the process of political evolution toward these types by choosing a lower tax rate than even he himself would want. The fact that $(i_0, _t)$ is decreasing in the state demonstrates, in fact, that the

incentive to "slow things up" intensifies the larger is the public sector. This is seen in the first diagram in Fig. 2. The second diagram displays transition dynamics using the notation $\binom{t}{t} = Q\binom{t}{t}$, $\binom{t}{t}$ for PA, $\binom{t}{t} = Q\binom{t}{t}$, $\binom{t}{t}$ for PE, and $\binom{t}{0}$, $\binom{t}{t} = Q\binom{t}{t}$, $\binom{t}{0}$, $\binom{t}{t}$ for Hypothetical PE, respectively.

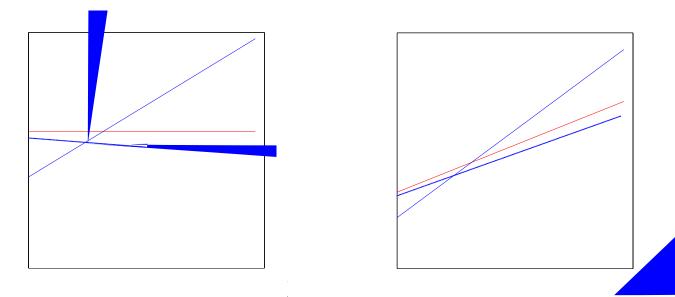


Figure 2: Monotone Policy Functions & Monotone Transition Dynamics in the PE and PA Equilibria.

This "Faustian" motive for gradualism described above can be identified in the leader's Euler equation. Using the parametric assumptions, the value function for the current leader i_t (but not for an arbitrary *i*) is

$$V(i_t, t; *) = \max_{a_t} \left\{ f(i)_t - \frac{a_t^2}{2} + V(i, t+1; *) \right\} \text{ subject to } t+1 = (1-d)_t + a_t$$
(10)

If (t_t) lies in the interior of the feasible policy space, then it satisfies the first-order condition¹⁶

$$0 = - *(_{t}) + D_{\omega_{t+1}} V (i_{t}, (1-d)_{t} + *(_{t}); *).$$
 (11)

The first term is clearly the marginal cost of an increase in government spending, while the second is the discounted marginal benefit in the future. Consider next period's decision from the point of view of the *current* decision maker i_t . Since next period's decision maker i_{t+1} is di erent from i_t , the decision next period induces a marginal distortion away from i_t 's optimal policy choice in t + 1. This distortion is given by

$$(i_{t, t+1}; *) = - *(_{t+1}) + D_{\omega_{t+2}} V (i_{t, t+1} + *(_{t+1}); *).$$
(12)

¹⁶Throughout the paper, the partial derivative of a function F(x, y) is expressed as $D_x F$.

Notice that the right-hand side of (12) is of the same form as (11), shifted one period ahead. Intuitively, because the government's capital stock $_t$ increases, power shifts toward policy makers who prefer higher levels of government spending. Hence, $(i_t, _{t+1}; *) < 0$ since i_{t+1} chooses a higher level of spending than i_t himself would choose in state $_{t+1}$.

Di erentiating the value function V in (10), substituting in the distortion equation (12), and iterating one period, the discounted marginal benefit $D_{\omega_{t+1}}V(i_t, t+1; *)$ of an increase in current investment can now be expressed as

$$D_{\omega_{t+1}}V(i_{t,t+1}; *) = [f(i_t) + (1-d) *(i_{t+1})] + [(K+1-d) (i_{t,t+1}; *)]$$
(13)

The marginal continuation value $D_{\omega_{t+1}}V$ can be decomposed into two e ects. The first bracketed term [·] on the right-hand side describes the direct e ect that current policy has on next period's state. This term also exists (is non-zero) under permanent authority, although the particular values di er between the two regimes. We refer to this di erence (between PE and PA) as the *reformation effect* since it describes the net direct incentive to increase public investment via changes in the marginal value of public sector capital. The reformation e ect does not include the "Faustian" distortion in incentives due to endogenous change in future leadership. This distortion is captured by the second bracketed term [·]. This term exists *only* in policy-endogenous decision problems. This distortion creates the "Burkean" incentive by all leaders to slow the evolution of political power as it moves away from the current leader. This is seen by the fact $(i_t, t+1; *) < 0$. Hence, we refer to it as the *political preservation effect*. General properties of both these e ects are described in the next Section.

Consider the decision of i_0 at date 0. His PE policy may be well below his PA policy, and it takes time before the PE path overtakes that of permanent authority. Part (iii) shows that this eventually happens (see Fig. 2). For t close to t_0 , the leader-type i_t is not so di erent from i_0 , and so the preference for conservative change may place $*(t_1)$ below (t_2) for a time.

Notice finally that the Faustian dynamics of states and leaders also moves monotonically. To see this, observe that if $_0$ is close to zero, it lies below the steady state. Hence, the equilibrium state transition $*(_t) \equiv Q(_t, *(_t))$ is increasing in the state, and, consequently, the equilibrium paths $\{_t\}$ and $\{i_t\}$ are increasing. This is illustrated in the second diagram in Fig. 2 which displays the comparison between the PE transition *, the hypothetical transition , and the PA transition .

So far, we have compared the PE to the PA regime. But this comparison can be applied to any two PE political institutions, one yielding more gradual change in political power than the other. This was illustrated in Figure 1.

Figure 3: Comparison of Two Political Institutions

Proposition 2 Consider two authority functions μ and $\tilde{\mu}$, each corresponding to a parameter pair $(_{0},)$ and $(_{0},)$ according to (9), and both of which satisfy the initial condition. Suppose $\geq \geq 0$ (and, consequently, $_{0} < _{0}$). Let * and \sim denote Policy-Endogenous (PE) equilibria under μ and $\tilde{\mu}$ respectively, and let * and \sim denote their corresponding (unique) steady states. Then $* \geq \sim iff_{0} < *$. Furthermore, if $_{0} < *$, then there exists a state $\hat{\mu}$ with $_{0} < \hat{\mu} < *$ such that

$$\begin{array}{cccc} * \left(\begin{array}{c} t \end{array}\right) & < & \widetilde{} \left(\begin{array}{c} t \end{array}\right) & whenever & _{0} \leq & _{t} < \widehat{} \\ * \left(\begin{array}{c} t \end{array}\right) & > & \widetilde{} \left(\begin{array}{c} t \end{array}\right) & whenever & _{t} > \widehat{}. \end{array}$$

whereas if $_0 > *$, then there exists a state $\hat{}$ with $* < \hat{} < _0$ such that

Recall that if $> \tilde{}$, then μ gives a faster *structural* evolution of political power, while $\tilde{\mu}$ is more gradual. The intuition is the same as in Part (iii) of Proposition 1. Decision makers respond to a more distorted political institution μ , by choosing *more* conservative responses. Initially, this Burkean incentive e ect outweighs the structural e ect from the authority functions. Hence, the more distorted iorititution μ initially produces a slower evolution of power and policy than the less di(it)2torted $\tilde{q}\tilde{p}$ eHowever, in the long run (including but not restricted to steady sitates), structural features take over and the more di(it)2torted polity produces a faster evolution of leaders and policies. This is illustrated in Figure 3.

3.4 Cyclical Faustian Dynamics

Now consider < 0, the case of countervailing distortion. Since μ slopes downward, increased public investment places power in more fiscally conservative citizen-types.

Proposition 3 Consider any authority function μ satisfying (9). If $0 > \geq -\frac{1-d}{\delta} - \frac{1+\delta}{1-\delta} \left[(1-d)^2 + \frac{3+\delta}{1+\delta} (1-d) + \frac{1}{\delta} \right]$, then the following hold.

- (i) There exist a decreasing affine Policy-Endogenous (PE) equilibrium policy rule $*(_t)$ and a nonincreasing, affine hypothetical rule $(i_0, _t)$, each of which are unique in the corresponding class of affine equilibria.
- (ii) The PE equilibrium path of states $\{ t \}$ converges to a unique steady state *. If $> -\frac{1-d}{\delta}$, then the convergence is monotonic. However, if $< -\frac{1-d}{\delta}$, then the economy follows a dampened cycle converging to * such that t < * if and only if t+1 > *. In either case, if 0 < *, then 0 < t implies $*(t) < (i_0, t) < (t)$, whereas is 0 > *, then t < 0 implies $*(t) > (i_0, t) < (t)$.

Faustian dynamics can therefore produce political cycles when the authority function is su ciently distorted downward. The intuition, roughly, is that the evolution of political power counters the evolution of public sector capital. Hence, when fiscally liberal types choose high expenditures, leading to increases in capital stock, this induces a steep drop in the index that determines the progressivity of the political type. More fiscally conservative types ther lower expenditures which, in turn, produce more liberal types, and so on.

Since μ slopes downward, then whenever the government's capital stock increases, politica power moves downward toward more fiscally conservative types. Consequently, the preservation e ect induces the initial, liberal, leader to decrease expenditures and hence *slow* the evolution of political authority as it moves downward. See Figures 4 and 5.

4 A General Monotone Model

This Section returns to the basic setup in Section 2. For tractability, we restrict attention to increasing authority functions. As before, we compare the PE equilibrium *, the PA

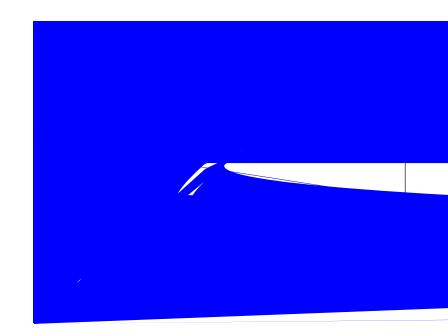


Figure 4: Cyclical Dynamics in the PE and PA Equilibria.

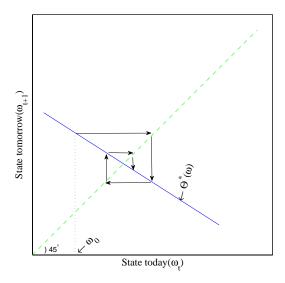


Figure 5: Cycles with Countervailing Bias

that the public investment model of the previous section is a special case of the general model examined here. All the results of this Section apply there as well. The proofs are in the Appendix.

Smoothness (di erentiability) plays a crucial role in our characterization. We use it to examine properties of the Euler equations, roughly following an approach dating back to Basar and Olsder (1982) for dynamic stochastic games.¹⁷ Results in Judd (2004) suggest that smoothness is a natural selection device when multiple equilibria exist. Formally, a PE equilibrium * and PA equilibrium are *smooth-limit equilibria* if (i) * and are di erentiable in the state; (ii) the resulting policies and *($_t$) and ($_t$) lie in the interior of the feasible policy space, and (iii) * and are the limit of smooth, finite horizon PE and PA equilibria, resp. Property (iii) is not necessary in the following characterization of the Euler

when i_0 holds power forever. The distortion equation (14) can be written as

$$D_{\omega_{t+1}}V_i = [D_{a_t}Q]^{-1} \cdot [(i, t; *) - D_{a_t}U_i]$$
(15)

Next, consider an arbitrary citizen-type $i \in I$ (not necessarily the leader) in the PE equilibrium. His continuation value function in state $_{t+1}$ is

V(i,

net incentive distortion due to the productivity di erences of a current policy choice in PE relative to PA. A higher level of future investment under * may increase one's incentive to invest more today. From Equation (13), the reformation e ect in the public investment model, for instance, is calculated to be (1 - d) (*($_{t+1}$) - ($_{t+1}$)). Since is stationary in that model, the reformation e ect is increasing in the state. Using Proposition 1, the reformation e ect is negative at states below a cuto $\hat{}$, and positive at states above it.

The second term, $P(i_t, t+1; *)$, is more closely identified with the Faustian trade o. Intuitively, it describes the marginal loss in i_t 's payo which is brought about when the policy induces a di erent, and clearly less desirable, political leader in the future. These future leaders choose policies that distort one's dynamic payo distortion away from its critical value. We refer to $P(i_t, t+1; *)$ as the *Preservation Effect* because, as we later show, it induces Burkean conservatism by decision makers. In the public investment model, the preservation e ect is given by (K + 1 - d) $(i_t, t+1; *)$ which is negative, a fact that we later show is robust.

The preservation e ect vanishes under the permanent authority rule due to the Envelope Theorem. Consequently, the Euler equation with i_0 as the permanent authority, is given by

$$D_{a_t} U_{i_0} + D_{a_t} Q \cdot R(i_0, t+1;) = 0$$
⁽²⁰⁾

where $_{t+1} = Q(_t, (_t))$ and $R(i_0, _{t+1}; \cdot)$ is defined by (19.a) and is evaluated by the PA equilibrium rather than by *.

The net e ect on continuation payo s of policy-endogenous power is given by

$$\underbrace{R(i, t+1; *) - R(i, t+1;)}_{R(i, t+1; *) - R(i, t+1;)} + \underbrace{P(i, t+1; *)}_{P(i, t+1; *)}$$
(21)

With the aid of a supermodularity assumption defined below, we use these terms to identify features of the PE equilibrium.

4.2 Supermodularity and Monotonicity

A central feature of the public investment model is monotonicity. In one case, political power shifts from lower to higher marginal valuation types, and this occurs when equilibrium policy rules are increasing in the state. In the second case, the paths exhibit cycles, with dampened oscillations between high and low valuation types. This occurs when equilibrium policy rules are decreasing in the state.

The monotonicity of the policy rule in these cases owes not so much to the specific functional forms, but (as we later show) rather to the fact that these functional forms are *supermodular*. This is no accident. Supermodularity is the more general version of a collection of commonly used assumptions (e.g., increasing di erences, strategic complements, etc.), that are generally used to establish monotonicity and/or well defined monotone comparative statics of endogenous decision rules. Here it is central in our e orts to characterize Faustian tradeo s. In smooth (di erentiable) models, the definition is straightforward. A smooth function $f: \mathbb{R}^m \to \mathbb{R}$ is supermodular (spm) i $D_{x_j}D_{x_k}f \ge 0$ for all j, k = 1, ..., m, and f is strictly spm if the inequality is strict.

First, define $a_t = L(t, t+1)$ implicitly from the transition equation Q(t, L(t, t+1)) = t+1. The function L(t, t+1) is interpreted as the policy cost of generating tomorrow's state t+1 given current state t.

(A1). Supermodularity (spm). The composite flow payo defined by $\tilde{u}(i, t, t+1) \equiv u(i, t, L(t, t+1))$ is supermodular in (i, t, t+1) and strictly supermodular in (i, t).

The composite flow payo captures both the static payo e ect of a change in policy and its future consequence through *L*. Hence, by placing joint restrictions on *u* and *Q* through the composite payo \tilde{u} , Assumption (A1) is weaker than separate spm assumptions on *u* and *Q*. Still, it is not unrestrictive. As with all spm assumptions, (A1) applies only to environments with su ciently strong complementarities between endogenous variables and exogenous parameters and between di erent sets of endogenous variables. It holds in most models of capital accumulation via taxation by a government since a government's current investment typically increases the productivity of its future investment. In the public investment model, for instance, (A1) is easily verified since $\tilde{u} = f(i)_{-t} - 1/2(_{-t+1} - (1 - d)_{-t})^2$.

On the other hand, spm often does not hold in situations where there are free rider problems as in, for instance, cases where public good provision is decentralized and voluntary. In such cases, an individual's marginal incentive to contribute diminishes the larger is the sum of others' contributions. The present model does not have this problem since public provision is centralized through a pivotal voting mechanism of one kind or another.

Theorem 1 Suppose that (A1) holds. Then in any smooth-limit Policy-Endogenous equilibrium *, citizen-type i's dynamic payoff, $U(i, t, a_t) + V(i, O(t, a_t); *)$, is supermodular in i and a_t .

Application of the Theorem, combined with standard results in voting theory (for instance, Gans and Smart (1996)) show that the Median Voter Theorem applies.¹⁹ Of course, for a given μ this median may be weighted as in Equation (8). Theorem 1 implies then that there exists a (possibly weighted) state-dependent voting rule such that the rule admits a pivotal voter $\mu(_t)$ in state $_t$.

¹⁹Assumption A1 implies that the payoff satisfies single crossing in i and a_t . Hence, the application of Gans' and Smart's result applies. See also Roberts (1998,1999) for another application of spm to a dynamic political economy model of club admissions.

A secondary consequence of spm is that it allows for a straightforward comparison of PE to the PA equilibrium based on the reformation e ect alone.

Theorem 2 Suppose Assumption (A1) holds. For each citizen-type *i* and each state $_t$, $*(_t) \ge (_t)$ iff $R(i,_t;_{t'}) \ge R(i,_t;_{t'})$.

Under (A1), a well ordered evolution of political authority exists in PE equilibria.

Theorem 3 Suppose (A1) holds. Let * be a smooth-limit Policy-Endogenous (PE) equilibrium and let *($_{t}$) $\equiv Q(_{t}, *(_{t}))$ be the associated PE equilibrium transition rule. Then, * is increasing, and for any state $_{t} \geq _{0}$, *($_{t}$) $\geq (i_{0}, _{t})$ with strict inequality if $_{t} \geq _{0}$.

The Theorem shows that the PE equilibrium transition rule is monotone in states, and the hypothetical PE rule is more conservative (in the natural order on policies) in every state than in the PE equilibrium. By itself, Theorem 3 does not say much about Faustian dynamics. However, under a simple initial condition, the following Corollary asserts that Faustian dynamics are monotone.

Corollary Suppose that, in addition to the assumptions of Theorem 3, $*(_0) > _0$. Then the Policy-Endogenous equilibrium paths of states $\{_t\}$ and leaders $\{i_t\}$ are increasing: $_{t+1} > _t$ and $i_{t+1} > i_t$.

The Corollary asserts that the PE equilibrium path of states and decision-making types is increasing provided it starts o that way. Hence, current leaders knowingly lose power to more progressive decision types, and this evolution continues until either a steady state is reached, or until the largest (most progressive) type acquires power.

Finally, spm is used to show that one critical feature of the public investment model holds quite broadly. So-called "Burkean" incentives for conservative decision making are embodied in the preservation e ect.

Theorem 4 Suppose (A1) holds. Let * be a smooth-limit PE equilibrium. For each state $_{t+1}$, $P(i, _{t+1}; *)$ is increasing in *i*, and

$$P(i_{t+1}; *) < 0 \quad if and only if \quad i < \mu(t_{t+1})$$
 (22)

In particular, the result implies that political leaders act in such a way as to o set, at least partially, the loss of political authority resulting from their policy decisions. Hence if the Faustian dynamics move toward more progressive leaders, then the preservation e ect pushes the current leader toward a more conservative policy. If the evolution is toward less progressive leaders, then the current leader acts more progressively. Combining Theorem 4 with the initial condition that $*(_0) > _0$ implies that the political preservation e ect is *negative* along the PE equilibrium path: $P(\mu(_t), _{t+1}; *) < 0$.

4.3 Steady States

By definition, the preservation e ect vanishes in any steady state $* = *(*) \equiv Q(*, *(*))$ of a PE equilibrium. The identity of the leader clearly does not change once * is reached. Consequently, the distortion adjusted Euler equation for the steady state leader $i^* = \mu(*)$ satisfies

$$D_{a_t} U(\mu(\ ^*),\ ^*,\ (\ ^*)) + D_{a_t} Q \cdot R(\mu(\ ^*),\ ^*;\ ^*) = 0$$
(23)

Using the composite payo $\tilde{u}(i, t, t+1)$ defined in Assumption (A1), Equation (23) reduces to

$$D_{\omega_{t+1}}\tilde{u}(\mu(*),*,*) + D_{\omega_t}\tilde{u}(\mu(*),*,*) = 0$$
(24)

The steady state equation (24) expresses a simple marginal tradeo in the steady state. In the public investment model, this tradeo was between present and future public sector capital. Similarly, $\circ = Q(\circ, (\circ))$ is a steady state in the PA equilibrium and the steady state equation is given by

$$D_{\omega_{t+1}}\tilde{U}(i_0, \circ, \circ) + D_{\omega_t}\tilde{U}(i_0, \circ, \circ) = 0$$

$$\tag{25}$$

Theorem 5 Suppose (A1) holds, and let * and be smooth-limit PE and PA equilibria, respectively. Then

- (i) * is a steady state of * iff (24) is satisfied, and \circ is a steady state of iff (25) is satisfied.
- (ii) The PE steady state * is unique if

$$D_{\omega_t}\tilde{u}(\mu(), ,) + D_{\omega_{t+1}}\tilde{u}(\mu(), ,)$$
 is decreasing in the state . (26)

and the PA steady state ° is unique if

$$D_{\omega_t}\tilde{u}(i_0, ,) + D_{\omega_{t+1}}\tilde{u}(i_0, ,)$$
 is decreasing in the state . (27)

(iii) Suppose that (27) holds for each *i*. Then $* > \circ$ iff $* > _0$.

Properties (i)-(iii) are all satisfied in the stylized model. Hence, the conclusions of the Theorem apply, indicating a degree of robustness of the steady state properties of that model.

Property (iii) is perhaps the most significant of the three. It relates the long-run in the PE equilibrium to the long run under permanent authority. It suggests that in the long run at least, progressive change in policy is not hindered by political expediency.

5 Related Literature and Discussion

The dynamic link between policy and power shows up in a small but growing number of papers. For purposes of relating these to the present paper, we found it most useful to separate them into two groups. One consists of models of one or two-period lived agents; the other consists of models with longer lived agents. As it turns out, there is an important di erence which our general model allows us to identify.

Milesi-Ferretti and Spolaore (1994) and Besley and Coate (1998) are early contributions to the first category of papers. They posit interesting two-period models in which a policy-maker's first period decision influences voter choices in the second. A "political failure", as Besley and Coate describe it, occurs when policy-endogenous loss of political control in the future lead to ine cient policy choices in the present.

Similarly, Bourguignon and Verdier (2000) explore a policy-endogenous mechanism that works through education and its e ect on political participation. Policies that change the level and distribution of education also change political participation across di erent groups. Dolmas and Hu man (2004)'s policy-endogenous mechanism works through immigration. Immigration policy is determined by majority vote, and immigration changes the identity of the median voter in the subsequent period. Campante (2007) examines a related mechanism that works through campaign contributions. In this case, policies that change income distribution alter the composition of contributions that, in turn, determine who is elected.

Hassler, et. al. (2003) investigate the evolution of the welfare state in a parametric overlapping-generations model. A majority vote determines the level of transfers to unsuccessful agents. Because the population sizes of di erent types are endogenously determined by individual investment decisions, their model can generate a shift of political power even with majority voting.²⁰

These studies highlight the broad array of mechanisms through which Faustian trade o s occur. We view the present model as complementary to these in that it suggests that equilibria in these frameworks have elements in common. Our findings also suggest some possible missing ingredients. Because agents in these models live (at most) two periods, the Faustian trade o is essentially one-shot. This means that a date t agent need not worry about the distortionary e ect his policy has on distribution of power beyond t + 1. In other words, these models have a reformation e ect but *no* preservation e ect. This is significant because the two e ects often work in opposite directions. Our results suggest that the time paths of models with and without the preservation e ect would be quite di erent.

Recent papers by Azzimonti (2005) and Ortega (2005) do have preservation e ects in mod-

²⁰Although, in their model, endogenous change in political power occurs mainly through private sector investment decisions rather than directly from current policies. A related model and policy-endogenous mechanism is studied in Hassler, et. al. (2005).

els with infinitely lived agents. Like Dolmas' and Hu man's model, Ortega (2005) studies a natural policy-endogenous mechanism in the form of immigration. Ceteris paribus, current residents want to admit immigrants with complementary skills. On the other hand such immigrants are future voters who will vote to admit future immigrants whose skills are substitutes to those of the current residents. Azzimonti (2005) posits an interesting model of dynamic ine ciency in government. An ine ciency arises because the dominant faction loses power to the other due to political shocks. She endogenizes the switching likelihoods between the two factions by introducing probabilistic voting. When the shocks to voters' ideological preferences for one group are asymmetric, then increases in public spending change voters' relative preferences between the groups, and so the identity of the pivotal voter changes as well.

Both these papers have something akin to both reformation and preservation e ects. Azzimonti, in fact, emphasizes a decomposition of motives in an Euler equation related to the one in our stylized model.²¹ However, the focus of these models is elsewhere, and both models' assumption of two political types/factions makes the Burkean conservatism of their decision makers di cult to identify. The present study therefore recasts these papers in a new light, allowing a clear view of subtle attributes that two seemingly di erent models have in common.

Clearly, one would not want to argue that all policy choices involve Faustian trade o s. In fact, a parallel literature has arisen that "un-couples" policy from political power by allowing the voters an explicit choice over political institutions. For instance, Acemoglu and Robinson (2000, 2001, 2006), Cervelatti, et. al. (2006), Jack and Laguno (2006a,b), Persson and Tabellini (2007) and Laguno (2008, 2009) all examine models of explicit institutional (de jure) choices by current elites or majorities as a way of reversing or mitigating the deleterious e ects of current policy on one's future political fortunes. Similarly, Roberts (1998, 1999) and Barbera, Maschler, and Shalev (BMS) (2001) un-couple the policy-political choice by placing attributes of a future pivotal or marginal voter directly in the preferences of the current voter. In this sense, the policy itself *is* the composition of political power.

These "un-coupling" models make sense when current elites have the flexibility to isolate or reverse the consequences of their policy choices. Our mechanism is appropriate when this flexibility is lacking.

This contrast is apparent in a recent model of Acemoglu and Robinson (2008). Building on an earlier framework laid out in Acemoglu, Johnson, and Robinson (2005), they model the policy decisions of an elite that explicitly preserve *de facto* political power when exogenous, *de jure* changes in the political system moves the country toward democracy. They identify "captured democracies" as those in which an elite's investments succeed in preserving power. The key di erence between their model and ours is that in our model, policies generate political change, while in theirs, policies are used by elites to undo (exogenous) political change. Acemoglu and Robinson look to 20th century Latin America for numerous instances

 $^{^{21}}$ Interestingly, her decomposition also includes exogenous inconsistency arising due to shocks rather than due to non-stationarity.

of captured democracies. On the other hand, the collapse of the Soviet Union, and the role of Glasnost in facilitating the change in power, suggests a Faustian trade o at work. A decision maker (Gorbachev) turned "Burkean" as he attempted moderate reforms that eventually lead

moderate their decisions in order to influence the future political evolution. This Burkean influence on individual incentives is a key consequence of the Faustian trade o . However, the Burkean influence must be weighed against the the dynamic change in political types when assessing the overall e ect on the equilibrium path. In the short-run the Burkean influence dominates when the PE equilibrium is compared with the PA model. In the long-run, however, the "type e ect" dominates. This indicates that there are critical features in the transition dynamics of the Faustian model that would not be evident by focusing only on steady state properties.

A few modeling choices warrant further discussion. First, decision makers are assumed to be exclusively policy-driven. Their desire for power is therefore purely instrumental. (This is in keeping with the original depiction of Faust as a well-intentioned character.) There are likely many historical examples of leaders who desire power for its own sake. It would not be di cult to incorporate "power-hungry" leaders into the model, however, this extension would be, in our view, rather prosaic.

Second, we omit stochastic shocks. Decision makers in the model choose not so much *whether* to lose power, but by *how much* and *to whom*. Consequently, we omit the case where leaders are uncertain about the political ramifications of their policies. As it turns out shocks do not fundamentally change the nature of the Faustian trade-o . They do introduce, however, risk aversion into the motives of the leader, and for this reason, would be a useful addition to future work.

On a related point, note that the model looks only at endogenous changes in political power. Of course, there may also be incentive e ects due to *exogenous* political change. These exogenous sources of change may be due to shocks, but they may also be built in to the authority rule. Exogenous sources of political change would move the analysis closer to traditional models of dynamically inconsistent policy choice — for instance hyperbolic — policy models²³ as well the famous fiscal policy models of Persson and Svensson (1989) and Alesina and Tabellini (1990). In these models a marginal e ect somewhat similar to the preservation e ect arises due to the conflict between current and future decision makers as calendar time changes the identity of the decision maker. Azzimonti's (2005) parametric model shows this explicitly.

Third, the stylized (parametric) model generates some fairly specific results on public investment, growth, and political change. Because the investment is assumed to be financed by lump sum taxation, no distortions arise other than that induced by the Faustian trade o. This is by design, given the focus of the paper. However, recent papers of Battaglini and Coate (2007, 2008) have made in-roads into our understanding of the dynamic political economy of distortionary taxation. At this stage there is still much work to do, and in light of the present results, explorations on the interaction between Faustian and tax distortions

 $^{^{23}}$ See Laibson (1997), Harris and Laibson (2001), Krusell, Kuruscu and Smith (2002), Krusell and Smith (2003), Amador (2003), Judd (2004).

seem well worth the e ort.

Finally, throughout the analysis, we keep the political institution exogenous in the analysis. We focus only on the de facto evolution of the political power within a stable (*de jure*) political institution. This allows us to examine the consequences of exogenous changes in political institutions. By construction, the framework does not answer the question of why a certain political institution is chosen and what determines the evolution of the de jure political institution. Future work could investigate the interaction of the policy-endogenous political power and policy-endogenous political institutions.

7 Appendix

Proof of Parts (i) in Propositions 1 and 2. For brevity, we combine Parts (i) of Propositions 1 and 2, since the argument does not depend per se on whether > 0 or < 0.

We first conjecture a solution * of the a ne form *() = (d - K) * + K where K is a constant (in), though we later show how it varies with . * is the steady state which depends on and $_0$. The conjecture is used to characterize both * and , the hypothetical rule. We establish a solution for coe cients (K, *) and establish uniqueness. We then characterize the case with = 0, and show that K = 0 there. This case gives us the PA equilibrium .

Step 1°. Verifying the Functional Forms. Using the a ne form as our "guess" the flow utility is

.

$$\begin{aligned} u(i, , a) &= f(i) -\frac{1}{2}((d-K)^{*} + K^{*})^{2} \\ &= f(i)^{*} -\frac{1}{2}d^{2}^{*2} + (f(i) - d^{*}K)(-^{*}) -\frac{1}{2}K^{2}(-^{*})^{2} \end{aligned}$$

For the purpose of solving for the equilibrium, we can drop the constant term. The continuation utility for an arbitrary *i* is

$$V(i, t; *) = \sum_{s=0}^{\infty} {}^{s} U(i, t+s, *(t+s)),$$

= $\sum_{s=0}^{\infty} {}^{s} \left[(f(i) - d *K) (t+s - *) - \frac{1}{2} K^{2} (t+s - *)^{2} \right]$
= $\frac{f(i) - d *K}{1 - (K + 1 - d)} (t-*) - \frac{1}{2} \frac{K^{2}}{1 - (K + 1 - d)^{2}} (t-*)^{2},$

where the last line follows from the fact that $_{t+s}-*=(K+1-d)^s(_t-*)$. The last equality above requires convergence of the infinite sum which, in turn, requires $K+1-d < \frac{1}{\delta}$ and

 $(K + 1 - d)^2 < \frac{1}{\delta}$ which combines to $K + 1 - d < \frac{1}{\sqrt{\delta}}$. The hypothetical problem confronting an arbitrary citizen-type *i* is

$$\max_{a_t} \{ u(i, t, a_t) + V(i, (1 - d) t + a_t; *) \}$$

which, when evaluated at the parametric assumptions produces the first-order condition

$$-a_{t} + \left[\frac{f(i) - d^{*}K}{1 - (K + 1 - d)} - \frac{K^{2}}{1 - (K + 1 - d)^{2}}((1 - d)_{t} + a_{t} - *)\right] = 0$$
(28)

The first-order condition (28) determines the hypothetical PE policy rule:

$$\begin{aligned} (i, t) &= \frac{1}{1 + \frac{\delta K^2}{1 - \delta (K + 1 - d)^2}} \left[\frac{f(i)}{1 - (K + 1 - d)} + \frac{\kappa \left(\frac{K}{1 - (K + 1 - d)^2} - \frac{d}{1 - (K + 1 - d)} \right) - \frac{K^2 (1 - d)}{1 - (K + 1 - d)^2} t \right] \end{aligned}$$

We return to the hypothetical equilibrium later. To determine the PE equilibrium, substitute $f(\mu()) = _0 + _$ in (29) to derive

$${}^{*}({}_{t}) = \frac{1}{1 + \frac{\delta K^{2}}{1 - \delta(K+1-d)^{2}}} \left[\frac{0}{1 - (K+1-d)} + {}^{*}K \left(\frac{K}{1 - (K+1-d)^{2}} - \frac{d}{1 - (K+1-d)} \right) \right]$$

$$+ \left(\frac{K^{2}(1-d)}{1 - (K+1-d)} - \frac{K^{2}(1-d)}{1 - (K+1-d)^{2}} \right) {}^{t} \right]$$

$$(30)$$

Hence, in order for $*(_t) = (d - K)^* + K_t$ to be a PE equilibrium, we must have

$$\mathcal{K} = \frac{\frac{\delta \kappa}{1 - \delta(K+1-d)} - \frac{\delta K^2 (1-d)}{1 - \delta(K+1-d)^2}}{1 + \frac{\delta K^2}{1 - \delta(K+1-d)^2}}, \text{ and}$$
(31)

$$(d - K)^{*} = \frac{\frac{\delta \kappa_{0}}{1 - \delta(K + 1 - d)} + \frac{K \left(\frac{K}{1 - \delta(K + 1 - d)^{2}} - \frac{d}{1 - \delta(K + 1 - d)}\right)}{1 + \frac{\delta K^{2}}{1 - \delta(K + 1 - d)^{2}}}.$$
 (32)

We therefore have two equations, and two unknowns, K and *. In what follows, we verify that there exists a unique pair (K, *) that satisfy (31) and (32).

Step 2°. Existence and uniqueness of the pair (K, *). The equation for K in (31) can be expressed as

$$K + \frac{K^2 (K + 1 - d)}{1 - (K + 1 - d)^2} - \frac{1}{1 - (K + 1 - d)} = 0.$$
(33)

Define $\hat{K} = K + 1 - d$. Multiply both sides of the equation (33) by

$$(1 - (K + 1 - d)^2)(1 - (K + 1 - d))$$

and after some messy algebra, the equation (33) becomes

$$F(\widehat{K}) \equiv (1-d)\widehat{K}^{3} + (-(2-d) - (1-d)^{2})\widehat{K}^{2} + (\frac{1}{-} + (1-d) + (1-d)^{2})\widehat{K} - (+\frac{1}{-} (1-d)) = 0.$$
(34)

Observe that the function F is of the form $F(\widehat{K}) = a_0() + a_1\widehat{K} + a_2()\widehat{K}^2 + a_3\widehat{K}^3$ (given the expression above, it should be clear that a_0 and a_2 vary with whereas a_1 and a_3 do not). We then have

$$F(1-d) = -(1-(1-d)^{2}),$$

$$F(1) = (1-)(d(\frac{1}{-}-1+d) -)$$

$$F(-1) = (+\frac{1-d}{-})(-1) - (1+)[(1-d)^{2} + \frac{3+}{1+}(1-d) + \frac{1}{-}]$$

$$F(0) = -(+\frac{1}{-}(1-d))$$

Suppose first that $0 < d(\frac{1}{\delta} - 1 + d)$ as required in Proposition 1. Then F(1 - d) < 0 and F(1) > 0. Then from the Intermediate Value Theorem, there exists a \hat{K}^* such that $1 - d < \hat{K}^* < 1$ (or equivalently $0 < K^* < d$) and $F(\hat{K}^*) = 0$.

Suppose next that $0 > -\frac{1-d}{\delta} - \frac{1+\delta}{1-\delta} \left[(1-d)^2 + \frac{3+\delta}{1+\delta} (1-d) + \frac{1}{\delta} \right]$ as required in Proposition 2. Then F(-1) < 0 and F(1-d) > 0. Once again, we apply the Intermediate Value Theorem to show that there exists a $-1 < \hat{K}^* < (1-d)$ (or equivalently $-(2-d) < K^* < 0$) such that $F(\hat{K}^*) = 0$.

To show uniqueness of the solution $\widehat{\mathcal{K}}^*$ in either the case of > 0 or < 0, it su ces to show that $F\left(\widehat{\mathcal{K}}\right)$ is concave, i.e., $F''\left(\widehat{\mathcal{K}}\right) = 2\left(a_2(\) + 3a_3\widehat{\mathcal{K}}\right) < 0$ for $-1 \le \widehat{\mathcal{K}} \le 1$. Towards this goal, it su ces to show that $a_2(\) + 3a_3 < 0$ (since $a_3 > 0$), i.e., $\left[-(2-d) - (1-d)^2 \right] + 3(1-d) < 0$. The latter is equivalent to the equation

$$\leq d\left(\frac{1}{2}-1+d\right)+2\left(\frac{1}{2}-1\right)(1-d)$$

which is always true since $\langle d(\frac{1}{\delta} - 1 + d)$. We conclude that $\hat{\mathcal{K}}^*$, and hence $\mathcal{K}^* = \hat{\mathcal{K}}^* - 1 + d$, is unique.

Having established a unique solution, K^* , we now solve for the steady state, *, from equation (32) and (33). After some algebra, we obtain

$$^{*} = \frac{0}{d\left(\frac{1}{\delta} - (1 - d)\right) - }.$$
(35)

We have therefore established a unique pair $(K^*, *)$ with K^* as the slope of * and * as the steady state satisfying (35). As the solution to F(K + 1 - d) = 0, notice that K^* varies with . We write $K^* = B($) to emphasize the dependence on . By the definition of F in (34), it is clear that B(0) = 0, which then yields an equation for the PA equilibrium . As for the hypothetical PE rule, , its solution form is also a ne. The coe cients of can be recovered from K^* and * evaluated at their respective solutions. The slope is non-positive and equal to zero i = 0 or d = 1.

Rest of the proof of Proposition 1. We now turn to Part (ii) of Proposition 1. To prove that convergence to the steady state is monotone, observe that

$${}^{*}({}_{t}) \equiv (1-d)_{t} + {}^{*}({}_{t}) = (1-d+K^{*})_{t} + (d-K^{*})^{*}.$$
(36)

Since $0 < K^* < d$, if $0 < < d(\frac{1}{\delta} - 1 + d)$, then convergence toward the steady state is monotonically increasing if $_0 < *$, and monotonically decreasing if $_0 > *$.

The rest of Part (ii) and (iii) can be readily verified from a straight-forward algebra. We omit the details and only give a sketch for the case $_0 < *$. The comparison of * and $(i_0,)$ follows directly from the a ne solution and the fact that $f(\mu())$ is increasing in . The comparison of and $(i_0,)$ follows from the higher slope of and the fact that $(i_0,) < (i_0,)$, where the latter can be checked easily from the solution. Given Part (ii) and the fact that the steady state is higher under PE equilibrium, Part (iii) then follows from the Intermediate Value Theorem.

Rest of the proof of Proposition 2. For Part (ii), an inspection of (36) reveals that $D_{\omega}^{*} > 0$ i $\hat{K}^{*} = 1 - d + K^{*} > 0$. Recalling the definition of F in the proof of Proposition 1, it is clear that F(0) < 0 and F(1 - d) > 0 if 0 > -(1 - d)/, and so the zero of F must satisfy $(1 - d) > \hat{K}^{*} > 0$. On the other hand, if $-(1 - d)/ > -\frac{1-d}{\delta} - \frac{1+\delta}{1-\delta} \left[(1 - d)^{2} + \frac{3+\delta}{1+\delta} (1 - d) + \frac{1}{\delta} \right]$, then F(0) > 0 and F(-1) < 0, and so the zero of F must satisfy $-1 < \hat{K}^{*} < 0$. In the latter case, the Faustian dynamics constitute a dampened cycle whereby, on-path, $t < t = \frac{1}{\delta} t =$

The rest of Part (ii) follows from a similar argument as the corresponding part of Proposition 1, with the main di erence being a negative . Hence we skip the details here.

Proof of Proposition 3. we restrict attention to the case where $_0 < *$, i.e., the initial state lies below the steady state. The logic when $_0 > *$ is symmetric.

It is easy to show that the steady state with is larger than that of \sim whenever $> \sim$. Given this and the a ne form of equilibrium, to show the final result it su ces to show that $*(_0) < \tilde{*}(_0)$. Let $K^* = B()$ and $\tilde{K}^* = B(\sim)$ denote the respective slope of PE equilibria * and $\tilde{*}$. We want to show that,

$$\left(d-\widetilde{K}^*\right)\frac{\widetilde{d}_0}{d\left(\frac{1}{\delta}-1+d\right)-\widetilde{d}_0}+\widetilde{K}^*_{0}>\left(d-K^*\right)\frac{0}{d\left(\frac{1}{\delta}-1+d\right)-\widetilde{d}_0}+K^*_{0},$$

which is equivalent to

$$\frac{d-B(\tilde{})}{d\left(\frac{1}{\delta}-1+d\right)-\tilde{}} < \frac{B()-B(\tilde{})}{-\tilde{}}.$$
(37)

In words, the last inequality requires that the slope between the points ($\tilde{}$, $B(\tilde{})$) and (, B()) is larger than that between ($\tilde{}$, $B(\tilde{})$) and ($d(\frac{1}{\delta} - 1 + d), d$) whenever $> \tilde{}$.

To proceed, we need to further characterize the properties of the solution $\mathcal{K}^* = B()$. But since \mathcal{K}^* and $\widehat{\mathcal{K}}^*$ di er only by a constant, we examine properties of the solution $\widehat{B}() \equiv \widehat{\mathcal{K}}^* = \widehat{B}() + 1 - d$ below. Recall that the unique solution $\widehat{\mathcal{K}}^* = \widehat{B}()$ is defined implicitly by the equation $F(\widehat{\mathcal{K}}) = 0$. Notice that, at the solution $\widehat{\mathcal{K}}^*$, $F(\widehat{\mathcal{K}}^*)$ satisfies

$$F'\left(\widehat{K}^*\right) = a_1 + 2a_2(\)\widehat{K}^* + 3a_3\left(\widehat{K}^*\right)^2 > 0,$$

$$F''\left(\widehat{K}^*\right) = 2\left(a_2(\) + 3a_3\widehat{K}^*\right) < 0.$$

Since $F'(\widehat{B}()) > 0$, $\widehat{B}()$ is continuously di erentiable from Implicit Function Theorem. In addition, we know that

$$\widehat{B}'() = \frac{1 - (\widehat{B}())^2}{a_1 + 2a_2()\widehat{B}() + 3a_3(\widehat{B}())^2} > 0$$

Take derivative again and after some algebra, we have

$$\widehat{B}''() = \frac{-2\left(1 - (\widehat{B}())^2\right)\left[3 \ a_3(\widehat{B}())^3 + 3 \ a_2()(\widehat{B}())^2 + (2 \ a_1 + 3a_3)\widehat{B}() + a_2()\right]}{\left[a_1 + 2a_2()\widehat{B}() + 3a_3(\widehat{B}())^2\right]^3}$$

Use the fact $a_0() + a_1\widehat{B}() + a_2()(\widehat{B}())^2 + a_3(\widehat{B}())^3 = 0$ to get

$$\widehat{B}''() = \frac{2\left(1 - (\widehat{B}())^2\right)(1 - (1 - d^2))\left[\widehat{B}() - b()\right]}{\left[a_1 + 2a_2()\widehat{B}() + 3a_3(\widehat{B}())^2\right]^3}$$

1

where $b() = \frac{-\delta d^2 - 2(1-\delta)d + (1-\delta) + 4\delta\kappa}{1-\delta(1-d^2)}$. Consequently, the sign of $\hat{B}''()$ is the same as that of $\hat{B}() - b()$.

In the following part, we show that there exists a unique $0 < - < d(\frac{1}{\delta} - 1 + d)$ such that (i) $\widehat{B}() > b()$ for < -; (ii) $\widehat{B}(-) = b(-)$ for = -; (iii) $\widehat{B}() < b()$ for > -. To start with, it is easy to see that $\widehat{B}(0) > b(0)$ and $\widehat{B}(d(\frac{1}{\delta} - 1 + d)) \le b(d(\frac{1}{\delta} - 1 + d))$. From the Intermediate Value Theorem there exists a - such that $\widehat{B}(-) = b(-)$.

Now we show that $\widehat{B}() > b()$ for < - and $\widehat{B}() < b()$ for > -. To show this, let

$$G() = F(b()) = a_3(b())^3 + a_2()(b())^2 + a_1b() + a_0().$$

The derivative of G() can be calculated as

$$G'() = [3a_3(b())^2 b'() + 2a_2() b() b'() + (b())^2 + a_1b'() - 1],$$

which equals to a quadratic form in b(),

$$\frac{6}{1-(1-d^2)} \left[\frac{1}{2} \left[d^2 - 4 d + 1 + 3 \right] (b())^2 - \left[d^2 - 2 (1 +) d + (3 +) \right] b() + \frac{1}{2} \left[d^2 - 4 d + 1 + 3 \right] \right].$$

To see the sign of G'(), first notice that the discriminant for the quadratic equation inside the bracket is

$${}^{2} \left[\begin{array}{cc} d^{2} - 2(1 +) d + (3 +) \right]^{2} - \left[\begin{array}{cc} d^{2} - 4 d + 1 + 3 \end{array} \right]^{2} \\ = \left(\begin{array}{cc} -1 \right) \left(1 - (1 - d)^{2} \right) < 0. \end{array}$$

Combine this with the positive quadratic coe cient, i.e., $d^2 - 4 d + 1 + 3 = d^2 + (1 - d) + 3 (1 - d) > 0$, we know that G'() > 0 for every . As a result, G() < 0 for < - and G() > 0 for > -. Therefore, $\widehat{B}() > b()$ for < - and $\widehat{B}() < b()$ for > -.

Translating back to our original coe cient, B(), we have thus established: B'() > 0, and B''() > 0 for < - and B''() < 0 for > -. In addition, it is easy to check that $B'(0) = \frac{1}{\frac{1}{\delta} - 1 + d}$, B(0) = 0, and $B(d(\frac{1}{\delta} - 1 + d)) = d$. These properties together implies that $B() \ge \frac{\kappa}{\frac{1}{\delta} - 1 + d}$, $\forall < d(\frac{1}{\delta - 1 + d})$. Geometrically, B() is an increasing function, convex below - and concave above -. In addition, B() lies above the line $\frac{\kappa}{\frac{1}{\delta} - 1 + d}$, with a unique tangent point at = 0.

Now we are ready to prove the inequality (37) from a geometric argument. Given the shape of B() established earlier, a simple graph shows that the inequality (37) always holds for $< d(\frac{1}{\delta-1+d})$. This concludes the proof of the Proposition.

Proof of Theorem 1. The proof is contained in the Proof of Theorem 3.

Proof of Theorem 2 From definition, $u(i, , a) = \tilde{u}(i, , Q(, a))$. Use this fact to replace u with \tilde{u} in equation (19.a), after some algebra it follows that

$$R(i, t, *) = D_{\omega_t} \tilde{u}(i, t, *(t)),$$
(38)

where *(t) = Q(t, *(t)) for any * (including permanent authority equilibrium ()). From (A1), R(i, t, *(t)) > R(i, t, (t)) if and only if Q(t, *(t)) > Q(t, (t)). From the monotonicity of Q(t, a) in a, Q(t, *(t)) > Q(t, (t)) if and only if *(t) > (t). This concludes the proof.

Proof of Theorem 3 For any arbitrary, smooth, strict supermodular continuation value $U(i, \cdot)$, define

$$H(i_{t,t}, a_{t,t}, U) = U(i_{t,t}, a_{t}) + U(i_{t}, Q(t_{t}, a_{t})).$$
(39)

 $H(i, t, a_t, U)$ is the payo function of a citizen-type *i* in state t when his continuation is defined by U(i, t+1). Let $(i, U) \in \arg \max_a H(i, t, a_t, U)$, and let $*(U, U) = (\mu(U), U, U)$.

To prove the monotonicity property, it is more convenient to work with a related representation as

$$\widetilde{H}(i, t, t+1, U) = \widetilde{U}(i, t, t+1) + U(i, t+1),$$
(40)

where $\tilde{u}(i, t, t+1)$ is defined in Assumption (A1). Define $(i, J) \in \arg \max_{\omega_{t+1}} \tilde{H}(i, t, t+1, U)$ and $*(J) = (\mu(J), J)$.

From the definition, it is immediate that $H(i, t, a_t, U) = \widetilde{H}(i, t, Q(t, a_t), U)$, (i, t, U) = Q(t, (i, t, U)) and $*(t, U) = Q(t, *(\mu(t), t, U))$. In addition, since $D_aQ(t, a) > 0$, $H(i, t, t, a_t, U)$ is supermodular in (i, a) if and only if $\widetilde{H}(i, t, t, t+1, U)$ is supermodular in (i, t, t+1).

We nit $\mathfrak{A}(\mu(i), \mathcal{H}(i), \mathcal{H})$

in (*i*,) by Assumption (A1). ²⁴ A simple backward induction argument establishes that $U_{t,T}$ satisfies strictly increasing di erences for all *t*. From the definition of the Smooth Limit Equilibrium, $||U_{t,T} - V(\cdot; *)|| \rightarrow 0$ as $T \rightarrow \infty$, and $V(\cdot; *)$, the infinite horizon PE equilibrium continuation value, has increasing di erences in *i* and . In fact, by repeating the step of the previous paragraph, it follows that $V(\cdot; *)$ must have strictly increasing di erences in *i* and .

Consequently, the solution (i, \cdot) must be strictly increasing in i, and so * must be strictly increasing in \cdot . This completes the proof of monotonicity of *.

This argument established *($_t$) = (μ ($_t$), $_t$) > (i_0 , $_t$) for all $_t$ > $_0$, or by definition

$$Q(t, *(t)) = *(t) > (i_0, t) = Q(t, (i_0, t)).$$
(41)

From the monotonicity of Q(, a) in a, we have $*(_t) > (i_0, _t)$.

Proof of Theorem 4. From definition, () = Q(, *()), with the derivative

$$D_{\omega} \quad () = D_{\omega}Q + D_{a}QD_{\omega} \quad ^{*}. \tag{42}$$

Combine equation (42) and (19.b) to get

$$P(i_{t,t+1}; *) = (D_{a_{t+1}}Q)^{-1} \cdot D_{\omega_{t+1}} * (i_{t+1}; *).$$
(43)

We know that, $D_{a_{t+1}}Q > 0$ and from the proof of Theorem 3, $D_{\omega_{t+1}} * > 0$. Therefore, P(i, t+1; *) < 0 follows if and only if (i, t+1; *) < 0.

Let $V^*(i, \cdot) \equiv V(i, \cdot; \cdot)$ be the equilibrium continuation value. Using the definition of H in (39) and that of the distortion function in (14), we have

$$D_{a_{t+1}}H(i_t, t_{t+1}, *(t_{t+1})), V^*) = (i_t, t_{t+1}; *)$$
(44)

Using the Proof of Theorem 3, which establishes strictly increasing di erences of H in i and a, it follows that (i, t+1; *) is increasing in i. It implies that whenever $i < i_{t+1} = \mu(t+1)$, then

$$(i_{t+1}; *) < (i_{t+1}, t+1; *) = 0$$
 (45)

where the latter equality follows from the type i_{t+1} 's first order condition in state t+1. This concludes the proof.

Proof of Theorem 5. Part (i). The permanent authority case follows from the standard argument. For the PE equilibrium, the necessity part is obvious. To see the su ciency,

²⁴In the last period T, $\underline{\omega}$ is chosen by the decision maker since there is no future return to the costly investment.

notice that the equation coincides with the steady state equation of the PA equilibrium with $i_0 = \mu(*)$. Since a PA decision maker faces fewer constraints than a PE one, if a PA authority decides to keep the state constant, it must be the optimal choice of a PE authority.

Part (ii) is obvious given Part (i).

We now prove Part (iii), i.e., $\circ < *$ if and only if $_0 < *$. Consider $i_0 = \mu(_0)$ and $i^* = \mu(*)$. From the strict increasing di erence between *i* and $(_{t, t+1})$, for each $(_{t, t+1}, _{t+2})$

$$D_{\omega_{t+1}} \widetilde{u} (i^*, t, t+1) + D_{\omega_{t+1}} \widetilde{u} (i^*, t+1, t+2) \\ > D_{\omega_{t+1}} \widetilde{u} (i_0, t, t+1) + D_{\omega_{t+1}} \widetilde{u} (i_0, t+1, t+2),$$

if and only if $i_0 < i^*$. Evaluate at $\begin{pmatrix} t, t+1, t+2 \end{pmatrix} = \begin{pmatrix} \circ, \circ, \circ \end{pmatrix}$ to get

$$D_{\omega_{t+1}}\widetilde{u}(i^{*}, \circ, \circ) + D_{\omega_{t+1}}\widetilde{u}(i^{*}, \circ, \circ)$$

$$> D_{\omega_{t+1}}\widetilde{u}(i_{0}, \circ, \circ) + D_{\omega_{t+1}}\widetilde{u}(i_{0}, \circ, \circ)$$

$$= 0$$

$$= D_{\omega_{t+1}}\widetilde{u}(i^{*}, *, *) + D_{\omega_{t+1}}\widetilde{u}(i^{*}, *, *),$$

if and only if $i^* > i_0$. Since $D_{\omega_{t+1}}\tilde{u}(i, ,) + D_{\omega_{t+1}}\tilde{u}(i, ,)$ is decreasing in , the inequality holds if and only if $\circ < *$. To summarize, we have just proven that $\circ < *$ if and only if $i_0 < i^*$, i.e., $_0 < *$.

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